Proof of Theorem 39

The theorem to be proved is

$\mathrm{C}(x, \mathrm{S}0, 0) = 0 \quad \vee \quad \mathrm{C}(x, \mathrm{S}0, 0) = \mathrm{S}0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\, (\mathrm{C}((x, \mathrm{S}0, 0))) = (0)] \quad \& \quad [\neg\, (\mathrm{C}((x, \mathrm{S}0, 0))) = (\mathrm{S}0)]]$

Special cases of the hypothesis and previous results:

0: $\neg\, \mathrm{C}((x, \mathrm{S}0, 0)) = 0$ from H:x

1: $\neg\, \mathrm{C}((x, \mathrm{S}0, 0)) = \mathrm{S}0$ from H:x

2: $0 = x \quad \vee \quad \mathrm{S}(\mathrm{P}x) = x$ from 22;x

3: $\mathrm{C}((0, \mathrm{S}0, 0)) = \mathrm{S}0$ from 33;S0;0;Px

4: $\mathrm{C}((\mathrm{S}(\mathrm{P}x), \mathrm{S}0, 0)) = 0$ from 33;S0;0;Px

Equality substitutions:

5: $\neg\, \mathrm{S}(\mathrm{P}x) = x \quad \vee \quad \neg\, \mathrm{C}((\mathrm{S}(\mathrm{P}x), \mathrm{S}0, 0)) = 0 \quad \vee \quad \mathrm{C}((x, \mathrm{S}0, 0)) = 0$

6: $\neg\ \ x = 0 \quad \vee \quad \mathrm{C}(((x), \mathrm{S}0, 0)) = \mathrm{S}0 \quad \vee \quad \neg\ \ \mathrm{C}(((0), \mathrm{S}0, 0)) = \mathrm{S}0$

Inferences:

7: $\neg\, \mathrm{S}(\mathrm{P}x) = x \quad \vee \quad \neg\, \mathrm{C}((\mathrm{S}(\mathrm{P}x), \mathrm{S}0, 0)) = 0$ by
0: $\neg\, \mathrm{C}((x, \mathrm{S}0, 0)) = 0$
5: $\neg\, \mathrm{S}(\mathrm{P}x) = x \quad \vee \quad \neg\, \mathrm{C}((\mathrm{S}(\mathrm{P}x), \mathrm{S}0, 0)) = 0 \quad \vee \quad \mathrm{C}((x, \mathrm{S}0, 0)) = 0$

8: $\neg\, 0 = x \quad \vee \quad \neg\, \mathrm{C}((0, \mathrm{S}0, 0)) = \mathrm{S}0$ by
1: $\neg\, \mathrm{C}((x, \mathrm{S}0, 0)) = \mathrm{S}0$
6: $\neg\, 0 = x \quad \vee \quad \mathrm{C}((x, \mathrm{S}0, 0)) = \mathrm{S}0 \quad \vee \quad \neg\, \mathrm{C}((0, \mathrm{S}0, 0)) = \mathrm{S}0$

9: $\neg\, 0 = x$ by
3: $\mathrm{C}((0, \mathrm{S}0, 0)) = \mathrm{S}0$
8: $\neg\, 0 = x \quad \vee \quad \neg\, \mathrm{C}((0, \mathrm{S}0, 0)) = \mathrm{S}0$

10: $\neg\, \mathrm{S}(\mathrm{P}x) = x$ by
4: $\mathrm{C}((\mathrm{S}(\mathrm{P}x), \mathrm{S}0, 0)) = 0$
7: $\neg\, \mathrm{S}(\mathrm{P}x) = x \quad \vee \quad \neg\, \mathrm{C}((\mathrm{S}(\mathrm{P}x), \mathrm{S}0, 0)) = 0$

11: $\mathrm{S}(\mathrm{P}x) = x$ by

9: $\neg\ 0 = x$

2: $0 = x \quad \vee \quad \mathrm{S}(\mathrm{P}x) = x$

12: QEA by

10: $\neg\ \mathrm{S}(\mathrm{P}x) = x$

11: $\mathrm{S}(\mathrm{P}x) = x$