Proof of Theorem 38

The theorem to be proved is

$C(x, 0, S0) = 0 \quad \vee \quad C(x, 0, S0) = S0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\, (C((x, 0, S0))) = (0)] \quad \& \quad [\neg\, (C((x, 0, S0))) = (S0)]]$

Special cases of the hypothesis and previous results:

0: $\neg\, C((x, 0, S0)) = 0$ from H:x

1: $\neg\, C((x, 0, S0)) = S0$ from H:x

2: $0 = x \quad \vee \quad S(Px) = x$ from 22;x

3: $C((0, 0, S0)) = 0$ from 33;0;S0;Px

4: $C((S(Px), 0, S0)) = S0$ from 33;0;S0;Px

Equality substitutions:

5: $\neg\, S(Px) = x \quad \vee \quad \neg\, C((S(Px), 0, S0)) = S0 \quad \vee \quad C((x, 0, S0)) = S0$

6: $\neg\; x = 0 \quad \vee \quad C(((x), 0, S0)) = 0 \quad \vee \quad \neg\; C(((0), 0, S0)) = 0$

Inferences:

7: $\neg\, 0 = x \quad \vee \quad \neg\, C((0, 0, S0)) = 0$ by
0: $\neg\, C((x, 0, S0)) = 0$
6: $\neg\, 0 = x \quad \vee \quad C((x, 0, S0)) = 0 \quad \vee \quad \neg\, C((0, 0, S0)) = 0$

8: $\neg\, S(Px) = x \quad \vee \quad \neg\, C((S(Px), 0, S0)) = S0$ by
1: $\neg\, C((x, 0, S0)) = S0$
5: $\neg\, S(Px) = x \quad \vee \quad \neg\, C((S(Px), 0, S0)) = S0 \quad \vee \quad C((x, 0, S0)) = S0$

9: $\neg\, 0 = x$ by
3: $C((0, 0, S0)) = 0$
7: $\neg\, 0 = x \quad \vee \quad \neg\, C((0, 0, S0)) = 0$

10: $\neg\, S(Px) = x$ by
4: $C((S(Px), 0, S0)) = S0$
8: $\neg\, S(Px) = x \quad \vee \quad \neg\, C((S(Px), 0, S0)) = S0$

11: $\mathrm{S}(\mathrm{P}x) = x$ by

9: $\neg\ 0 = x$

2: $0 = x \quad \vee \quad \mathrm{S}(\mathrm{P}x) = x$

12: QEA by

10: $\neg\ \mathrm{S}(\mathrm{P}x) = x$

11: $\mathrm{S}(\mathrm{P}x) = x$