Proof of Theorem 36

The theorem to be proved is

$C(x, 0, S0) = 0 \quad \leftrightarrow \quad x = 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(C((x, 0, S0))) = (0) \quad \vee \quad \neg\ (C((x, 0, S0))) = (0)] \quad \& \quad [(C((x, 0, S0))) = (0) \quad \vee \quad (x) = (0)] \quad \& \quad [\neg\ (x) = (0) \quad \vee \quad \neg\ (C((x, 0, S0))) = (0)] \quad \& \quad [\neg\ (x) = (0) \quad \vee \quad (x) = (0)]]$

Special cases of the hypothesis and previous results:

0: $C((x, 0, S0)) = 0 \quad \vee \quad 0 = x$ from H:x

1: $\neg\ 0 = x \quad \vee \quad \neg\ C((x, 0, S0)) = 0$ from H:x

2: $0 = x \quad \vee \quad S(Px) = x$ from 22;x

3: $C((0, 0, S0)) = 0$ from 33;0;S0;Px

4: $C((S(Px), 0, S0)) = S0$ from 33;0;S0;Px

5: $\neg\ S0 = 0$ from 3;0

Equality substitutions:

6: $\neg\ C((x, 0, S0)) = 0 \quad \vee \quad \neg\ C((x, 0, S0)) = S0 \quad \vee \quad 0 = S0$

7: $\neg\ S(Px) = x \quad \vee \quad \neg\ C((S(Px), 0, S0)) = S0 \quad \vee \quad C((x, 0, S0)) = S0$

8: $\neg\ x = 0 \quad \vee \quad C(((x), 0, S0)) = 0 \quad \vee \quad \neg\ C(((0), 0, S0)) = 0$

Inferences:

9: $\neg\ 0 = x \quad \vee \quad C((x, 0, S0)) = 0$ by

3: $C((0, 0, S0)) = 0$

8: $\neg\ 0 = x \quad \vee \quad C((x, 0, S0)) = 0 \quad \vee \quad \neg\ C((0, 0, S0)) = 0$

10: $\neg\ S(Px) = x \quad \vee \quad C((x, 0, S0)) = S0$ by

4: $C((S(Px), 0, S0)) = S0$

7: $\neg\ S(Px) = x \quad \vee \quad \neg\ C((S(Px), 0, S0)) = S0 \quad \vee \quad C((x, 0, S0)) = S0$

11: $\neg\ C((x, 0, S0)) = 0 \quad \vee \quad \neg\ C((x, 0, S0)) = S0$ by

5: $\neg\ S0 = 0$

6: $\neg\ C((x, 0, S0)) = 0 \quad \vee \quad \neg\ C((x, 0, S0)) = S0 \quad \vee \quad S0 = 0$

CLAIM: $\neg\, 0 = x$ Suppose not. Then

12: $0 = x$

13: $\neg\, \mathrm{C}((x, 0, \mathrm{S}0)) = 0$ by

12: $0 = x$

1: $\neg\, 0 = x \quad \vee \quad \neg\, \mathrm{C}((x, 0, \mathrm{S}0)) = 0$

14: $\mathrm{C}((x, 0, \mathrm{S}0)) = 0$ by

12: $0 = x$

9: $\neg\, 0 = x \quad \vee \quad \mathrm{C}((x, 0, \mathrm{S}0)) = 0$

15: $\neg\, 0 = x$ The CLAIM is proved, and 12–14 will not be used after this:

13: $\neg\, \mathrm{C}((x, 0, \mathrm{S}0)) = 0$

14: $\mathrm{C}((x, 0, \mathrm{S}0)) = 0$

16: $\mathrm{C}((x, 0, \mathrm{S}0)) = 0$ by

15: $\neg\, 0 = x$

0: $\mathrm{C}((x, 0, \mathrm{S}0)) = 0 \quad \vee \quad 0 = x$

17: $\mathrm{S}(\mathrm{P}x) = x$ by

15: $\neg\, 0 = x$

2: $0 = x \quad \vee \quad \mathrm{S}(\mathrm{P}x) = x$

18: $\neg\, \mathrm{C}((x, 0, \mathrm{S}0)) = \mathrm{S}0$ by

16: $\mathrm{C}((x, 0, \mathrm{S}0)) = 0$

11: $\neg\, \mathrm{C}((x, 0, \mathrm{S}0)) = 0 \quad \vee \quad \neg\, \mathrm{C}((x, 0, \mathrm{S}0)) = \mathrm{S}0$

19: $\mathrm{C}((x, 0, \mathrm{S}0)) = \mathrm{S}0$ by

17: $\mathrm{S}(\mathrm{P}x) = x$

10: $\neg\, \mathrm{S}(\mathrm{P}x) = x \quad \vee \quad \mathrm{C}((x, 0, \mathrm{S}0)) = \mathrm{S}0$

20: QEA by

18: $\neg\, \mathrm{C}((x, 0, \mathrm{S}0)) = \mathrm{S}0$

19: $\mathrm{C}((x, 0, \mathrm{S}0)) = \mathrm{S}0$