## Proof of Theorem 35

The theorem to be proved is

$\mathrm{C}(x, y, z) \neq z \quad \rightarrow \quad x = 0 \quad \& \quad \mathrm{C}(x, y, z) = y$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\,(\mathrm{C}((x, y, z))) = (z)] \quad \& \quad [\neg\,(x) = (0) \quad \vee \quad \neg\,(\mathrm{C}((x, y, z))) = (y)]]$

### Special cases of the hypothesis and previous results:

0: $\neg\,\mathrm{C}((x, y, z)) = z$ from H:$x$:$y$:$z$

1: $\neg\,0 = x \quad \vee \quad \neg\,\mathrm{C}((x, y, z)) = y$ from H:$x$:$y$:$z$

2: $0 = x \quad \vee \quad \mathrm{C}((x, y, z)) = z$ from 34;$x$;$y$;$z$

3: $\mathrm{C}((0, y, z)) = y$ from 33;$y$;$z$

### Equality substitutions:

4: $\neg\,0 = x \quad \vee \quad \neg\,\mathrm{C}((0, y, z)) = y \quad \vee \quad \mathrm{C}((x, y, z)) = y$

### Inferences:

5: $0 = x$ by
0: $\neg\,\mathrm{C}((x, y, z)) = z$
2: $0 = x \quad \vee \quad \mathrm{C}((x, y, z)) = z$

6: $\neg\,0 = x \quad \vee \quad \mathrm{C}((x, y, z)) = y$ by
3: $\mathrm{C}((0, y, z)) = y$
4: $\neg\,0 = x \quad \vee \quad \neg\,\mathrm{C}((0, y, z)) = y \quad \vee \quad \mathrm{C}((x, y, z)) = y$

7: $\neg\,\mathrm{C}((x, y, z)) = y$ by
5: $0 = x$
1: $\neg\,0 = x \quad \vee \quad \neg\,\mathrm{C}((x, y, z)) = y$

8: $\mathrm{C}((x, y, z)) = y$ by
5: $0 = x$
6: $\neg\,0 = x \quad \vee \quad \mathrm{C}((x, y, z)) = y$

9: $QEA$ by
7: $\neg\,\mathrm{C}((x, y, z)) = y$
8: $\mathrm{C}((x, y, z)) = y$