Proof of Theorem 34

The theorem to be proved is

$x \neq 0 \quad \rightarrow \quad \mathrm{C}(x, y, z) = z$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\,(x) = (0)] \quad \& \quad [\neg\,(\mathrm{C}((x, y, z))) = (z)]]$

Special cases of the hypothesis and previous results:

0: $\neg\, 0 = x$ from H:x:y:z

1: $\neg\, \mathrm{C}((x, y, z)) = z$ from H:x:y:z

2: $0 = x \quad \vee \quad \mathrm{S}(\mathrm{P}x) = x$ from 22;x

3: $\mathrm{C}((\mathrm{S}(\mathrm{P}x), y, z)) = z$ from 33;y;z;Px

Equality substitutions:

4: $\neg\, \mathrm{S}(\mathrm{P}x) = x \quad \vee \quad \neg\, \mathrm{C}((\mathrm{S}(\mathrm{P}x), y, z)) = z \quad \vee \quad \mathrm{C}((x, y, z)) = z$

Inferences:

5: $\mathrm{S}(\mathrm{P}x) = x$ by
0: $\neg\, 0 = x$
2: $0 = x \quad \vee \quad \mathrm{S}(\mathrm{P}x) = x$

6: $\neg\, \mathrm{S}(\mathrm{P}x) = x \quad \vee \quad \neg\, \mathrm{C}((\mathrm{S}(\mathrm{P}x), y, z)) = z$ by
1: $\neg\, \mathrm{C}((x, y, z)) = z$
4: $\neg\, \mathrm{S}(\mathrm{P}x) = x \quad \vee \quad \neg\, \mathrm{C}((\mathrm{S}(\mathrm{P}x), y, z)) = z \quad \vee \quad \mathrm{C}((x, y, z)) = z$

7: $\neg\, \mathrm{S}(\mathrm{P}x) = x$ by
3: $\mathrm{C}((\mathrm{S}(\mathrm{P}x), y, z)) = z$
6: $\neg\, \mathrm{S}(\mathrm{P}x) = x \quad \vee \quad \neg\, \mathrm{C}((\mathrm{S}(\mathrm{P}x), y, z)) = z$

8: QEA by
5: $\mathrm{S}(\mathrm{P}x) = x$
7: $\neg\, \mathrm{S}(\mathrm{P}x) = x$