Proof of Theorem 30

The theorem to be proved is

$x - y = 0 \quad \& \quad y - x = 0 \quad \rightarrow \quad x = y$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x - y) = (0)] \quad \& \quad [(y - x) = (0)] \quad \& \quad [\neg\, (x) = (y)]]$

Special cases of the hypothesis and previous results:

0: $x - y = 0$ from H:x:y

1: $y - x = 0$ from H:x:y

2: $\neg\, y = x$ from H:x:y

3: $y = x \quad \vee \quad \neg\, y - x = 0 \quad \vee \quad \neg\, x - y = 0$ from 29;x;y

Inferences:

4: $y = x \quad \vee \quad \neg\, y - x = 0$ by
0: $x - y = 0$
3: $y = x \quad \vee \quad \neg\, y - x = 0 \quad \vee \quad \neg\, x - y = 0$

5: $y = x$ by
1: $y - x = 0$
4: $y = x \quad \vee \quad \neg\, y - x = 0$

6: QEA by
2: $\neg\, y = x$
5: $y = x$