## Proof of Theorem 29i

The theorem to be proved is

$$[x = y \quad \vee \quad y - x \neq 0 \quad \vee \quad x - y \neq 0] \quad \rightarrow \quad [\mathrm{S}x = y \quad \vee \quad y - \mathrm{S}x \neq 0 \quad \vee \quad \mathrm{S}x - y \neq 0]$$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x) = (y) \quad \vee \quad \neg\ (y - x) = (0) \quad \vee \quad \neg\ (x - y) = (0)] \quad \& \quad [\neg\ (\mathrm{S}x) = (y)] \quad \& \quad [(y - (\mathrm{S}x)) = (0)] \quad \& \quad [((\mathrm{S}x) - y) = (0)]]$

### Special cases of the hypothesis and previous results:

0: $y = x \quad \vee \quad \neg\ y - x = 0 \quad \vee \quad \neg\ x - y = 0$ from H:$x$:$y$

1: $\neg\ \mathrm{S}x = y$ from H:$x$:$y$

2: $y - (\mathrm{S}x) = 0$ from H:$x$:$y$

3: $(\mathrm{S}x) - y = 0$ from H:$x$:$y$

4: $\neg\ (\mathrm{S}x) - x = 0$ from 21;$x$

5: $y - x = 0 \quad \vee \quad \mathrm{S}x = y \quad \vee \quad \neg\ y - (\mathrm{S}x) = 0$ from 27;$y$;$x$

6: $x - y = 0 \quad \vee \quad \neg\ (\mathrm{S}x) - y = 0$ from 28;$x$;$y$

### Equality substitutions:

7: $\neg\ y = x \quad \vee \quad \neg\ (\mathrm{S}x) - y = 0 \quad \vee \quad (\mathrm{S}x) - x = 0$

### Inferences:

8: $y - x = 0 \quad \vee \quad \neg\ y - (\mathrm{S}x) = 0$ by
1: $\neg\ \mathrm{S}x = y$
5: $y - x = 0 \quad \vee \quad \mathrm{S}x = y \quad \vee \quad \neg\ y - (\mathrm{S}x) = 0$

9: $y - x = 0$ by
2: $y - (\mathrm{S}x) = 0$
8: $y - x = 0 \quad \vee \quad \neg\ y - (\mathrm{S}x) = 0$

10: $x - y = 0$ by
3: $(\mathrm{S}x) - y = 0$
6: $x - y = 0 \quad \vee \quad \neg\ (\mathrm{S}x) - y = 0$

11: $\neg\, y = x \quad \vee \quad (\mathrm{S}x) - x = 0$ by

3: $(\mathrm{S}x) - y = 0$

7: $\neg\, y = x \quad \vee \quad \neg\, (\mathrm{S}x) - y = 0 \quad \vee \quad (\mathrm{S}x) - x = 0$

12: $\neg\, y = x$ by

4: $\neg\, (\mathrm{S}x) - x = 0$

11: $\neg\, y = x \quad \vee \quad (\mathrm{S}x) - x = 0$

13: $y = x \quad \vee \quad \neg\, x - y = 0$ by

9: $y - x = 0$

0: $y = x \quad \vee \quad \neg\, y - x = 0 \quad \vee \quad \neg\, x - y = 0$

14: $y = x$ by

10: $x - y = 0$

13: $y = x \quad \vee \quad \neg\, x - y = 0$

15: $QEA$ by

12: $\neg\, y = x$

14: $y = x$