

Proof of Theorem 29b

The theorem to be proved is

$$0 = y \vee y - 0 \neq 0 \vee 0 - y \neq 0$$

Suppose the theorem does not hold. Then, with the variables held fixed,

$$(H) \quad [[\neg (0) = (y)] \ \& \ [(y - 0) = (0)] \ \& \ [(0 - y) = (0)]]$$

Special cases of the hypothesis and previous results:

$$0: \quad \neg 0 = y \quad \text{from } H:y$$

$$1: \quad y - 0 = 0 \quad \text{from } H:y$$

$$2: \quad y - 0 = y \quad \text{from } \underline{17};y$$

Equality substitutions:

$$3: \quad \neg y - 0 = 0 \vee \neg y - 0 = y \vee 0 = y$$

Inferences:

$$4: \quad \neg y - 0 = 0 \vee \neg y - 0 = y \quad \text{by}$$

$$0: \quad \neg 0 = y$$

$$3: \quad \neg y - 0 = 0 \vee \neg y - 0 = y \vee 0 = y$$

$$5: \quad \neg y - 0 = y \quad \text{by}$$

$$1: \quad y - 0 = 0$$

$$4: \quad \neg y - 0 = 0 \vee \neg y - 0 = y$$

$$6: \quad QEA \quad \text{by}$$

$$2: \quad y - 0 = y$$

$$5: \quad \neg y - 0 = y$$