Proof of Theorem 294

The theorem to be proved is

x ends with x & x ends with ϵ & x begins with x & x begins with ϵ

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\,(x)$ ends with (x) $\vee$ $\neg\,(x)$ ends with (ϵ) $\vee$ $\neg\,(x)$ begins with (x) $\vee$ $\neg\,(x)$ begins with $(\epsilon)]]$

Special cases of the hypothesis and previous results:

0: $\neg\, x$ ends with x $\vee$ $\neg\, x$ ends with ϵ $\vee$ $\neg\, x$ begins with x $\vee$ $\neg\, x$ begins with ϵ from H:x

1: $\epsilon \oplus x = x$ from 194;x

2: $x \oplus \epsilon = x$ from 196;x

3: $\epsilon \oplus x$ ends with x from 289;ϵ;x

4: $x \oplus \epsilon$ ends with ϵ from 289;x;ϵ

5: $\epsilon \oplus x$ begins with ϵ from 291;ϵ;x

6: $x \oplus \epsilon$ begins with x from 291;x;ϵ

Equality substitutions:

7: $\neg\, \epsilon \oplus x = x$ $\vee$ $\neg\, \epsilon \oplus x$ ends with x $\vee$ x ends with x

8: $\neg\, \epsilon \oplus x = x$ $\vee$ $\neg\, \epsilon \oplus x$ begins with ϵ $\vee$ x begins with ϵ

9: $\neg\, x \oplus \epsilon = x$ $\vee$ $\neg\, x \oplus \epsilon$ ends with ϵ $\vee$ x ends with ϵ

10: $\neg\, x \oplus \epsilon = x$ $\vee$ $\neg\, x \oplus \epsilon$ begins with x $\vee$ x begins with x

Inferences:

11: $\neg\, \epsilon \oplus x$ ends with x $\vee$ x ends with x by
1: $\epsilon \oplus x = x$
7: $\neg\, \epsilon \oplus x = x$ $\vee$ $\neg\, \epsilon \oplus x$ ends with x $\vee$ x ends with x

12: $\neg\, \epsilon \oplus x$ begins with ϵ $\vee$ x begins with ϵ by
1: $\epsilon \oplus x = x$
8: $\neg\, \epsilon \oplus x = x$ $\vee$ $\neg\, \epsilon \oplus x$ begins with ϵ $\vee$ x begins with ϵ

13: $\neg\, x \oplus \epsilon$ ends with ϵ $\vee$ x ends with ϵ by

2: $x \oplus \epsilon = x$

9: $\neg\, x \oplus \epsilon = x$ $\vee$ $\neg\, x \oplus \epsilon$ ends with ϵ $\vee$ x ends with ϵ

14: $\neg\, x \oplus \epsilon$ begins with x $\vee$ x begins with x by

2: $x \oplus \epsilon = x$

10: $\neg\, x \oplus \epsilon = x$ $\vee$ $\neg\, x \oplus \epsilon$ begins with x $\vee$ x begins with x

15: x ends with x by

3: $\epsilon \oplus x$ ends with x

11: $\neg\, \epsilon \oplus x$ ends with x $\vee$ x ends with x

16: x ends with ϵ by

4: $x \oplus \epsilon$ ends with ϵ

13: $\neg\, x \oplus \epsilon$ ends with ϵ $\vee$ x ends with ϵ

17: x begins with ϵ by

5: $\epsilon \oplus x$ begins with ϵ

12: $\neg\, \epsilon \oplus x$ begins with ϵ $\vee$ x begins with ϵ

18: x begins with x by

6: $x \oplus \epsilon$ begins with x

14: $\neg\, x \oplus \epsilon$ begins with x $\vee$ x begins with x

19: $\neg\, x$ ends with ϵ $\vee$ $\neg\, x$ begins with x $\vee$ $\neg\, x$ begins with ϵ by

15: x ends with x

0: $\neg\, x$ ends with x $\vee$ $\neg\, x$ ends with ϵ $\vee$ $\neg\, x$ begins with x $\vee$ $\neg\, x$ begins with ϵ

20: $\neg\, x$ begins with x $\vee$ $\neg\, x$ begins with ϵ by

16: x ends with ϵ

19: $\neg\, x$ ends with ϵ $\vee$ $\neg\, x$ begins with x $\vee$ $\neg\, x$ begins with ϵ

21: $\neg\, x$ begins with x by

17: x begins with ϵ

20: $\neg\, x$ begins with x $\vee$ $\neg\, x$ begins with ϵ

22: QEA by

18: x begins with x

21: $\neg\, x$ begins with x