Proof of Theorem 293

The theorem to be proved is

y ends with $c \quad \rightarrow \quad x \oplus y$ ends with c

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(y)$ ends with $(c)] \quad \& \quad [\neg\, (x \oplus y)$ ends with $(c)]]$

Special cases of the hypothesis and previous results:

0: y ends with c from H:y:c:x

1: $\neg\, x \oplus y$ ends with c from H:y:c:x

2: $\neg\, y$ ends with $c \quad \vee \quad a \oplus c = y$ from 288$^{\rightarrow}$;y;c:a

3: $x \oplus (a \oplus c) = (x \oplus a) \oplus c$ from 183;x;a;c

4: $(x \oplus a) \oplus c$ ends with c from 289;$x \oplus a$;c

Equality substitutions:

5: $\neg\, a \oplus c = y \quad \vee \quad \neg\, x \oplus (a \oplus c) = (x \oplus a) \oplus c \quad \vee \quad x \oplus (y) = (x \oplus a) \oplus c$

6: $\neg\, (x \oplus a) \oplus c = x \oplus y \quad \vee \quad \neg\, (x \oplus a) \oplus c$ ends with $c \quad \vee \quad x \oplus y$ ends with c

Inferences:

7: $a \oplus c = y$ by
0: y ends with c
2: $\neg\, y$ ends with $c \quad \vee \quad a \oplus c = y$

8: $\neg\, (x \oplus a) \oplus c = x \oplus y \quad \vee \quad \neg\, (x \oplus a) \oplus c$ ends with c by
1: $\neg\, x \oplus y$ ends with c
6: $\neg\, (x \oplus a) \oplus c = x \oplus y \quad \vee \quad \neg\, (x \oplus a) \oplus c$ ends with $c \quad \vee \quad x \oplus y$ ends with c

9: $\neg\, a \oplus c = y \quad \vee \quad (x \oplus a) \oplus c = x \oplus y$ by
3: $x \oplus (a \oplus c) = (x \oplus a) \oplus c$
5: $\neg\, a \oplus c = y \quad \vee \quad \neg\, x \oplus (a \oplus c) = (x \oplus a) \oplus c \quad \vee \quad (x \oplus a) \oplus c = x \oplus y$

10: $\neg\, (x \oplus a) \oplus c = x \oplus y$ by
4: $(x \oplus a) \oplus c$ ends with c
8: $\neg\, (x \oplus a) \oplus c = x \oplus y \quad \vee \quad \neg\, (x \oplus a) \oplus c$ ends with c

11: $(x \oplus a) \oplus c = x \oplus y$ by

7: $a \oplus c = y$

9: $\neg\ a \oplus c = y \quad \vee \quad (x \oplus a) \oplus c = x \oplus y$

12: QEA by

10: $\neg\ (x \oplus a) \oplus c = x \oplus y$

11: $(x \oplus a) \oplus c = x \oplus y$