Proof of Theorem 292

The theorem to be proved is

x begins with $a \quad \rightarrow \quad x \oplus y$ begins with a

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x)$ begins with $(a)] \quad \& \quad [\neg\,(x \oplus y)$ begins with $(a)]]$

Special cases of the hypothesis and previous results:

0: x begins with a from H:x:a:y

1: $\neg\, x \oplus y$ begins with a from H:x:a:y

2: $\neg\, x$ begins with $a \quad \vee \quad a \oplus c = x$ from 290$^{\rightarrow}$;x;a:c

3: $a \oplus (c \oplus y) = (a \oplus c) \oplus y$ from 183;a;c;y

4: $a \oplus (c \oplus y)$ begins with a from 291;a;$c \oplus y$

Equality substitutions:

5: $\neg\, a \oplus c = x \quad \vee \quad \neg\, a \oplus (c \oplus y) = (a \oplus c) \oplus y \quad \vee \quad a \oplus (c \oplus y) = (x) \oplus y$

6: $\neg\, a \oplus (c \oplus y) = x \oplus y \quad \vee \quad \neg\, a \oplus (c \oplus y)$ begins with $a \quad \vee \quad x \oplus y$ begins with a

Inferences:

7: $a \oplus c = x$ by

0: x begins with a

2: $\neg\, x$ begins with $a \quad \vee \quad a \oplus c = x$

8: $\neg\, a \oplus (c \oplus y) = x \oplus y \quad \vee \quad \neg\, a \oplus (c \oplus y)$ begins with a by

1: $\neg\, x \oplus y$ begins with a

6: $\neg\, a \oplus (c \oplus y) = x \oplus y \quad \vee \quad \neg\, a \oplus (c \oplus y)$ begins with $a \quad \vee \quad x \oplus y$ begins with a

9: $\neg\, a \oplus c = x \quad \vee \quad a \oplus (c \oplus y) = x \oplus y$ by

3: $a \oplus (c \oplus y) = (a \oplus c) \oplus y$

5: $\neg\, a \oplus c = x \quad \vee \quad \neg\, a \oplus (c \oplus y) = (a \oplus c) \oplus y \quad \vee \quad a \oplus (c \oplus y) = x \oplus y$

10: $\neg\, a \oplus (c \oplus y) = x \oplus y$ by

4: $a \oplus (c \oplus y)$ begins with a

8: $\neg\, a \oplus (c \oplus y) = x \oplus y \quad \vee \quad \neg\, a \oplus (c \oplus y)$ begins with a

11: $a \oplus (c \oplus y) = x \oplus y$ by

7: $a \oplus c = x$

9: $\neg\, a \oplus c = x \quad \vee \quad a \oplus (c \oplus y) = x \oplus y$

12: QEA by

10: $\neg\, a \oplus (c \oplus y) = x \oplus y$

11: $a \oplus (c \oplus y) = x \oplus y$