Proof of Theorem 291

The theorem to be proved is

 $a \oplus c$ begins with a

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg (a \oplus c) \text{ begins with } (a)]]$

Special cases of the hypothesis and previous results:

- 0: $\neg a \oplus c$ begins with a from H:a:c
- 1: $c \leq a \oplus c$ from 277;a;c
- 2: $a \oplus c$ begins with $a \lor \neg c \preceq a \oplus c \lor \neg a \oplus c = a \oplus c$ from $290 < ; a \oplus c; a; c$
- 3: $a \oplus c = a \oplus c$ from $\underline{5}; a \oplus c$

Inferences:

- 4: $\neg c \leq a \oplus c \quad \lor \quad \neg a \oplus c = a \oplus c \quad \text{by}$
 - 0: $\neg a \oplus c$ begins with a
 - 2: $a \oplus c$ begins with $a \lor \neg c \preceq a \oplus c \lor \neg a \oplus c = a \oplus c$
- 5: $\neg a \oplus c = a \oplus c$ by
 - 1: $c \leq a \oplus c$
 - 4: $\neg c \leq a \oplus c \quad \lor \quad \neg a \oplus c = a \oplus c$
- 6: QEA by
 - 3: $a \oplus c = a \oplus c$
 - 5: $\neg a \oplus c = a \oplus c$