Proof of Theorem 28i

The theorem to be proved is

$$[x - y \neq 0 \quad \rightarrow \quad \mathrm{S}x - y \neq 0] \quad \rightarrow \quad [x - \mathrm{S}y \neq 0 \quad \rightarrow \quad \mathrm{S}x - \mathrm{S}y \neq 0]$$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x-y) = (0) \quad \vee \quad \neg\, ((\mathrm{S}x)-y) = (0)] \quad \& \quad [\neg\, (x-(\mathrm{S}y)) = (0)] \quad \& \quad [((\mathrm{S}x)-(\mathrm{S}y)) = (0)]]$

Special cases of the hypothesis and previous results:

0: $\neg\, x - (\mathrm{S}y) = 0$ from H:x:y

1: $(\mathrm{S}x) - (\mathrm{S}y) = 0$ from H:x:y

2: $(\mathrm{S}x) - (\mathrm{S}y) = x - y$ from 18;x;y

3: $\mathrm{P}(x - y) = x - (\mathrm{S}y)$ from 17;x;y

4: $\mathrm{P}0 = 0$ from 16

Equality substitutions:

5: $\neg\, x - y = 0 \quad \vee \quad \mathrm{P}(x - y) = 0 \quad \vee \quad \neg\, \mathrm{P}(0) = 0$

6: $\neg\, (\mathrm{S}x) - (\mathrm{S}y) = 0 \quad \vee \quad \neg\, (\mathrm{S}x) - (\mathrm{S}y) = x - y \quad \vee \quad 0 = x - y$

7: $\neg\, \mathrm{P}(x - y) = x - (\mathrm{S}y) \quad \vee \quad \neg\, \mathrm{P}(x - y) = 0 \quad \vee \quad x - (\mathrm{S}y) = 0$

Inferences:

8: $\neg\, \mathrm{P}(x - y) = x - (\mathrm{S}y) \quad \vee \quad \neg\, \mathrm{P}(x - y) = 0$ by
0: $\neg\, x - (\mathrm{S}y) = 0$
7: $\neg\, \mathrm{P}(x - y) = x - (\mathrm{S}y) \quad \vee \quad \neg\, \mathrm{P}(x - y) = 0 \quad \vee \quad x - (\mathrm{S}y) = 0$

9: $\neg\, (\mathrm{S}x) - (\mathrm{S}y) = x - y \quad \vee \quad x - y = 0$ by
1: $(\mathrm{S}x) - (\mathrm{S}y) = 0$
6: $\neg\, (\mathrm{S}x) - (\mathrm{S}y) = 0 \quad \vee \quad \neg\, (\mathrm{S}x) - (\mathrm{S}y) = x - y \quad \vee \quad x - y = 0$

10: $x - y = 0$ by
2: $(\mathrm{S}x) - (\mathrm{S}y) = x - y$
9: $\neg\, (\mathrm{S}x) - (\mathrm{S}y) = x - y \quad \vee \quad x - y = 0$

11: $\neg\, \mathrm{P}(x - y) = 0$ by

3: $\mathrm{P}(x - y) = x - (\mathrm{S}y)$

8: $\neg\, \mathrm{P}(x - y) = x - (\mathrm{S}y) \quad \vee \quad \neg\, \mathrm{P}(x - y) = 0$

12: $\neg\, x - y = 0 \quad \vee \quad \mathrm{P}(x - y) = 0$ by

4: $\mathrm{P}0 = 0$

5: $\neg\, x - y = 0 \quad \vee \quad \mathrm{P}(x - y) = 0 \quad \vee \quad \neg\, \mathrm{P}0 = 0$

13: $\mathrm{P}(x - y) = 0$ by

10: $x - y = 0$

12: $\neg\, x - y = 0 \quad \vee \quad \mathrm{P}(x - y) = 0$

14: QEA by

11: $\neg\, \mathrm{P}(x - y) = 0$

13: $\mathrm{P}(x - y) = 0$