Proof of Theorem 28b

The theorem to be proved is

$x - 0 \neq 0 \quad \rightarrow \quad \mathrm{S}x - 0 \neq 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\,(x - 0) = (0)] \quad \& \quad [((\mathrm{S}x) - 0) = (0)]]$

Special cases of the hypothesis and previous results:

0: $(\mathrm{S}x) - 0 = 0$ from H:x

1: $(\mathrm{S}x) - 0 = \mathrm{S}x$ from 17;Sx

2: $\neg\,\mathrm{S}x = 0$ from 3;x

Equality substitutions:

3: $\neg\,(\mathrm{S}x) - 0 = 0 \quad \vee \quad \neg\,(\mathrm{S}x) - 0 = \mathrm{S}x \quad \vee \quad 0 = \mathrm{S}x$

Inferences:

4: $\neg\,(\mathrm{S}x) - 0 = \mathrm{S}x \quad \vee \quad \mathrm{S}x = 0$ by

0: $(\mathrm{S}x) - 0 = 0$

3: $\neg\,(\mathrm{S}x) - 0 = 0 \quad \vee \quad \neg\,(\mathrm{S}x) - 0 = \mathrm{S}x \quad \vee \quad \mathrm{S}x = 0$

5: $\mathrm{S}x = 0$ by

1: $(\mathrm{S}x) - 0 = \mathrm{S}x$

4: $\neg\,(\mathrm{S}x) - 0 = \mathrm{S}x \quad \vee \quad \mathrm{S}x = 0$

6: QEA by

2: $\neg\,\mathrm{S}x = 0$

5: $\mathrm{S}x = 0$