Proof of Theorem 278

The theorem to be proved is

$\text{Length}\,\underline{0} = 1 \quad \& \quad \text{Length}\,\underline{1} = 1$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\,(\text{Length}\underline{0}) = (1) \quad \vee \quad \neg\,(\text{Length}\underline{1}) = (1)]]$

Special cases of the hypothesis and previous results:

0: $\neg\,\text{Length}\underline{0} = 1 \quad \vee \quad \neg\,\text{Length}\underline{1} = 1$ from H

1: $\text{Length}\text{the}\epsilon = 0$ from 259;ϵ

2: $\text{Length}(\epsilon \oplus \underline{0}) = \text{S}(\text{Length}\text{the}\epsilon)$ from 259;ϵ

3: $\text{Length}(\epsilon \oplus \underline{1}) = \text{S}(\text{Length}\text{the}\epsilon)$ from 259;ϵ

4: $\epsilon \oplus \underline{0} = \underline{0}$ from 194;$\underline{0}$

5: $\epsilon \oplus \underline{1} = \underline{1}$ from 194;$\underline{1}$

6: $\text{S}0 = 1$ from 115

Equality substitutions:

7: $\neg\,\text{Length}\text{the}\epsilon = 0 \quad \vee \quad \text{S}(\text{Length}\text{the}\epsilon) = 1 \quad \vee \quad \neg\,\text{S}(0) = 1$

8: $\neg\,\epsilon \oplus \underline{0} = \underline{0} \quad \vee \quad \neg\,\text{Length}(\epsilon \oplus \underline{0}) = \text{S}(\text{Length}\text{the}\epsilon) \quad \vee \quad \text{Length}(\underline{0}) = \text{S}(\text{Length}\text{the}\epsilon)$

9: $\neg\,\epsilon \oplus \underline{1} = \underline{1} \quad \vee \quad \neg\,\text{Length}(\epsilon \oplus \underline{1}) = \text{S}(\text{Length}\text{the}\epsilon) \quad \vee \quad \text{Length}(\underline{1}) = \text{S}(\text{Length}\text{the}\epsilon)$

10: $\neg\,\text{S}(\text{Length}\text{the}\epsilon) = \text{Length}\underline{0} \quad \vee \quad \neg\,\text{S}(\text{Length}\text{the}\epsilon) = 1 \quad \vee \quad \text{Length}\underline{0} = 1$

11: $\neg\,\text{S}(\text{Length}\text{the}\epsilon) = \text{Length}\underline{1} \quad \vee \quad \neg\,\text{S}(\text{Length}\text{the}\epsilon) = 1 \quad \vee \quad \text{Length}\underline{1} = 1$

Inferences:

12: $\text{S}(\text{Length}\text{the}\epsilon) = 1 \quad \vee \quad \neg\,\text{S}0 = 1$ by

1: $\text{Length}\text{the}\epsilon = 0$

7: $\neg\,\text{Length}\text{the}\epsilon = 0 \quad \vee \quad \text{S}(\text{Length}\text{the}\epsilon) = 1 \quad \vee \quad \neg\,\text{S}0 = 1$

13: $\neg\,\epsilon \oplus \underline{0} = \underline{0} \quad \vee \quad \text{S}(\text{Length}\text{the}\epsilon) = \text{Length}\underline{0}$ by

2: $\text{Length}(\epsilon \oplus \underline{0}) = \text{S}(\text{Length}\text{the}\epsilon)$

8: $\neg\,\epsilon \oplus \underline{0} = \underline{0} \quad \vee \quad \neg\,\text{Length}(\epsilon \oplus \underline{0}) = \text{S}(\text{Length}\text{the}\epsilon) \quad \vee \quad \text{S}(\text{Length}\text{the}\epsilon) = \text{Length}\underline{0}$

14: $\neg\ \epsilon \oplus \underline{1} = \underline{1} \quad \vee \quad \mathrm{S}(\mathrm{Length}\epsilon) = \mathrm{Length}\underline{1}$ by

3: $\mathrm{Length}(\epsilon \oplus \underline{1}) = \mathrm{S}(\mathrm{Length}\epsilon)$

9: $\neg\ \epsilon \oplus \underline{1} = \underline{1} \quad \vee \quad \neg\ \mathrm{Length}(\epsilon \oplus \underline{1}) = \mathrm{S}(\mathrm{Length}\epsilon) \quad \vee \quad \mathrm{S}(\mathrm{Length}\epsilon) = \mathrm{Length}\underline{1}$

15: $\mathrm{S}(\mathrm{Length}\epsilon) = \mathrm{Length}\underline{0}$ by

4: $\epsilon \oplus \underline{0} = \underline{0}$

13: $\neg\ \epsilon \oplus \underline{0} = \underline{0} \quad \vee \quad \mathrm{S}(\mathrm{Length}\epsilon) = \mathrm{Length}\underline{0}$

16: $\mathrm{S}(\mathrm{Length}\epsilon) = \mathrm{Length}\underline{1}$ by

5: $\epsilon \oplus \underline{1} = \underline{1}$

14: $\neg\ \epsilon \oplus \underline{1} = \underline{1} \quad \vee \quad \mathrm{S}(\mathrm{Length}\epsilon) = \mathrm{Length}\underline{1}$

17: $\mathrm{S}(\mathrm{Length}\epsilon) = 1$ by

6: $\mathrm{S}0 = 1$

12: $\mathrm{S}(\mathrm{Length}\epsilon) = 1 \quad \vee \quad \neg\ \mathrm{S}0 = 1$

18: $\neg\ \mathrm{S}(\mathrm{Length}\epsilon) = 1 \quad \vee \quad \mathrm{Length}\underline{0} = 1$ by

15: $\mathrm{S}(\mathrm{Length}\epsilon) = \mathrm{Length}\underline{0}$

10: $\neg\ \mathrm{S}(\mathrm{Length}\epsilon) = \mathrm{Length}\underline{0} \quad \vee \quad \neg\ \mathrm{S}(\mathrm{Length}\epsilon) = 1 \quad \vee \quad \mathrm{Length}\underline{0} = 1$

19: $\neg\ \mathrm{S}(\mathrm{Length}\epsilon) = 1 \quad \vee \quad \mathrm{Length}\underline{1} = 1$ by

16: $\mathrm{S}(\mathrm{Length}\epsilon) = \mathrm{Length}\underline{1}$

11: $\neg\ \mathrm{S}(\mathrm{Length}\epsilon) = \mathrm{Length}\underline{1} \quad \vee \quad \neg\ \mathrm{S}(\mathrm{Length}\epsilon) = 1 \quad \vee \quad \mathrm{Length}\underline{1} = 1$

20: $\mathrm{Length}\underline{0} = 1$ by

17: $\mathrm{S}(\mathrm{Length}\epsilon) = 1$

18: $\neg\ \mathrm{S}(\mathrm{Length}\epsilon) = 1 \quad \vee \quad \mathrm{Length}\underline{0} = 1$

21: $\mathrm{Length}\underline{1} = 1$ by

17: $\mathrm{S}(\mathrm{Length}\epsilon) = 1$

19: $\neg\ \mathrm{S}(\mathrm{Length}\epsilon) = 1 \quad \vee \quad \mathrm{Length}\underline{1} = 1$

22: $\neg\ \mathrm{Length}\underline{1} = 1$ by

20: $\mathrm{Length}\underline{0} = 1$

0: $\neg\ \mathrm{Length}\underline{0} = 1 \quad \vee \quad \neg\ \mathrm{Length}\underline{1} = 1$

23: QEA by

21: $\mathrm{Length}\underline{1} = 1$

22: $\neg\ \mathrm{Length}\underline{1} = 1$