

## Proof of Theorem 276

The theorem to be proved is

$$x \oplus y \preceq y \oplus x$$

Suppose the theorem does not hold. Then, with the variables held fixed,

$$(H) \quad [[\neg (x \oplus y) \preceq (y \oplus x)]]$$

### Special cases of the hypothesis and previous results:

- 0:  $\neg x \oplus y \preceq y \oplus x$       from   H: $x:y$
- 1:  $(\text{Length}x) + (\text{Length}y) = \text{Length}(x \oplus y)$       from   [260](#); $x;y$
- 2:  $(\text{Length}y) + (\text{Length}x) = \text{Length}(y \oplus x)$       from   [260](#); $y;x$
- 3:  $(\text{Length}y) + (\text{Length}x) = (\text{Length}x) + (\text{Length}y)$       from   [98](#); $\text{Length}x;\text{Length}y$
- 4:  $(\text{Length}x) + (\text{Length}y) \leq (\text{Length}x) + (\text{Length}y)$       from   [60](#); $\text{Length}x) + (\text{Length}y$
- 5:  $x \oplus y \preceq y \oplus x \quad \vee \quad \neg \text{Length}(x \oplus y) \leq \text{Length}(y \oplus x)$       from   [264](#)<sup><</sup>; $x \oplus y;y \oplus x$

### Equality substitutions:

- 6:  $\neg (\text{Length}x) + (\text{Length}y) = \text{Length}(x \oplus y) \quad \vee \quad \neg (\text{Length}y) + (\text{Length}x) =$   
 $(\text{Length}x) + (\text{Length}y) \quad \vee \quad (\text{Length}y) + (\text{Length}x) = \text{Length}(x \oplus y)$
- 7:  $\neg (\text{Length}x) + (\text{Length}y) = \text{Length}(x \oplus y) \quad \vee \quad \neg (\text{Length}x) + (\text{Length}y) \leq$   
 $(\text{Length}x) + (\text{Length}y) \quad \vee \quad \text{Length}(x \oplus y) \leq \text{Length}(x \oplus y)$
- 8:  $\neg (\text{Length}y) + (\text{Length}x) = \text{Length}(y \oplus x) \quad \vee \quad \neg (\text{Length}y) + (\text{Length}x) =$   
 $\text{Length}(x \oplus y) \quad \vee \quad \text{Length}(y \oplus x) = \text{Length}(x \oplus y)$
- 9:  $\neg \text{Length}(y \oplus x) = \text{Length}(x \oplus y) \quad \vee \quad \text{Length}(x \oplus y) \leq \text{Length}(y \oplus x) \quad \vee$   
 $\neg \text{Length}(x \oplus y) \leq \text{Length}(x \oplus y)$

### Inferences:

- 10:  $\neg \text{Length}(x \oplus y) \leq \text{Length}(y \oplus x)$       by
  - 0:  $\neg x \oplus y \preceq y \oplus x$
  - 5:  $x \oplus y \preceq y \oplus x \quad \vee \quad \neg \text{Length}(x \oplus y) \leq \text{Length}(y \oplus x)$
- 11:  $\neg (\text{Length}y) + (\text{Length}x) = (\text{Length}x) + (\text{Length}y) \quad \vee \quad (\text{Length}y) + (\text{Length}x) =$   
 $\text{Length}(x \oplus y)$       by

- 1:  $(\text{Length}x) + (\text{Length}y) = \text{Length}(x \oplus y)$
- 6:  $\neg (\text{Length}x) + (\text{Length}y) = \text{Length}(x \oplus y) \quad \vee \quad \neg (\text{Length}y) + (\text{Length}x) = (\text{Length}x) + (\text{Length}y) \quad \vee \quad (\text{Length}y) + (\text{Length}x) = \text{Length}(x \oplus y)$
- 12:  $\neg (\text{Length}x) + (\text{Length}y) \leq (\text{Length}x) + (\text{Length}y) \quad \vee \quad \text{Length}(x \oplus y) \leq \text{Length}(x \oplus y)$   
by
- 1:  $(\text{Length}x) + (\text{Length}y) = \text{Length}(x \oplus y)$
- 7:  $\neg (\text{Length}x) + (\text{Length}y) = \text{Length}(x \oplus y) \quad \vee \quad \neg (\text{Length}x) + (\text{Length}y) \leq (\text{Length}x) + (\text{Length}y) \quad \vee \quad \text{Length}(x \oplus y) \leq \text{Length}(x \oplus y)$
- 13:  $\neg (\text{Length}y) + (\text{Length}x) = \text{Length}(x \oplus y) \quad \vee \quad \text{Length}(y \oplus x) = \text{Length}(x \oplus y)$   
by
- 2:  $(\text{Length}y) + (\text{Length}x) = \text{Length}(y \oplus x)$
- 8:  $\neg (\text{Length}y) + (\text{Length}x) = \text{Length}(y \oplus x) \quad \vee \quad \neg (\text{Length}y) + (\text{Length}x) = \text{Length}(x \oplus y) \quad \vee \quad \text{Length}(y \oplus x) = \text{Length}(x \oplus y)$
- 14:  $(\text{Length}y) + (\text{Length}x) = \text{Length}(x \oplus y)$  by
- 3:  $(\text{Length}y) + (\text{Length}x) = (\text{Length}x) + (\text{Length}y)$
- 11:  $\neg (\text{Length}y) + (\text{Length}x) = (\text{Length}x) + (\text{Length}y) \quad \vee \quad (\text{Length}y) + (\text{Length}x) = \text{Length}(x \oplus y)$
- 15:  $\text{Length}(x \oplus y) \leq \text{Length}(x \oplus y)$  by
- 4:  $(\text{Length}x) + (\text{Length}y) \leq (\text{Length}x) + (\text{Length}y)$
- 12:  $\neg (\text{Length}x) + (\text{Length}y) \leq (\text{Length}x) + (\text{Length}y) \quad \vee \quad \text{Length}(x \oplus y) \leq \text{Length}(x \oplus y)$
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- 10:  $\neg \text{Length}(x \oplus y) \leq \text{Length}(y \oplus x)$
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- 19: *QEA* by
- 17:  $\text{Length}(y \oplus x) = \text{Length}(x \oplus y)$

18:  $\neg \text{Length}(y \oplus x) = \text{Length}(x \oplus y)$