Proof of Theorem 275

The theorem to be proved is

$x \preceq y \quad \& \quad y \preceq z \quad \rightarrow \quad x \preceq z$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x) \preceq (y)] \quad \& \quad [(y) \preceq (z)] \quad \& \quad [\neg\, (x) \preceq (z)]]$

Special cases of the hypothesis and previous results:

0: $x \preceq y$ from H:x:y:z

1: $y \preceq z$ from H:x:y:z

2: $\neg\, x \preceq z$ from H:x:y:z

3: $\neg\, x \preceq y \quad \vee \quad \text{Length}x \leq \text{Length}y$ from $264^{\rightarrow}$;x;y

4: $\neg\, y \preceq z \quad \vee \quad \text{Length}y \leq \text{Length}z$ from $264^{\rightarrow}$;y;z

5: $\neg\, \text{Length}x \leq \text{Length}y \quad \vee \quad \neg\, \text{Length}y \leq \text{Length}z \quad \vee \quad \text{Length}x \leq \text{Length}z$ from 73;Length$x$;Length$y$;Length$z$

6: $x \preceq z \quad \vee \quad \neg\, \text{Length}x \leq \text{Length}z$ from $264^{\leftarrow}$;x;z

Inferences:

7: $\text{Length}x \leq \text{Length}y$ by

0: $x \preceq y$

3: $\neg\, x \preceq y \quad \vee \quad \text{Length}x \leq \text{Length}y$

8: $\text{Length}y \leq \text{Length}z$ by

1: $y \preceq z$

4: $\neg\, y \preceq z \quad \vee \quad \text{Length}y \leq \text{Length}z$

9: $\neg\, \text{Length}x \leq \text{Length}z$ by

2: $\neg\, x \preceq z$

6: $x \preceq z \quad \vee \quad \neg\, \text{Length}x \leq \text{Length}z$

10: $\neg\, \text{Length}y \leq \text{Length}z \quad \vee \quad \text{Length}x \leq \text{Length}z$ by

7: $\text{Length}x \leq \text{Length}y$

5: $\neg\, \text{Length}x \leq \text{Length}y \quad \vee \quad \neg\, \text{Length}y \leq \text{Length}z \quad \vee \quad \text{Length}x \leq \text{Length}z$

11: $\text{Length}x \leq \text{Length}z$ by

8: $\text{Length}y \leq \text{Length}z$

10: $\neg\, \text{Length}y \leq \text{Length}z \quad \vee \quad \text{Length}x \leq \text{Length}z$

12: QEA by

9: $\neg$ Length$x \leq$ Length$z$

11: Length$x \leq$ Length$z$