Proof of Theorem 274

The theorem to be proved is

$\epsilon \preceq x \quad \& \quad x \preceq x$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\,(\epsilon) \preceq (x) \quad \vee \quad \neg\,(x) \preceq (x)]]$

Special cases of the hypothesis and previous results:

0: $\neg\,\epsilon \preceq x \quad \vee \quad \neg\,x \preceq x$ from H:x

1: $\epsilon \preceq x \quad \vee \quad \neg\,\mathrm{Length}\epsilon \leq \mathrm{Length}x$ from 264$^{\Leftarrow}$;ϵ;x

2: $x \preceq x \quad \vee \quad \neg\,\mathrm{Length}x \leq \mathrm{Length}x$ from 264$^{\Leftarrow}$;x;x

3: $\mathrm{Length}\epsilon = 0$ from 259

4: $0 \leq \mathrm{Length}x$ from 58;Length$x$

5: $\mathrm{Length}x \leq \mathrm{Length}x$ from 60;Length$x$

Equality substitutions:

6: $\neg\,\mathrm{Length}\epsilon = 0 \quad \vee \quad \mathrm{Length}\epsilon \leq \mathrm{Length}x \quad \vee \quad \neg\,0 \leq \mathrm{Length}x$

Inferences:

7: $\mathrm{Length}\epsilon \leq \mathrm{Length}x \quad \vee \quad \neg\,0 \leq \mathrm{Length}x$ by
3: $\mathrm{Length}\epsilon = 0$
6: $\neg\,\mathrm{Length}\epsilon = 0 \quad \vee \quad \mathrm{Length}\epsilon \leq \mathrm{Length}x \quad \vee \quad \neg\,0 \leq \mathrm{Length}x$

8: $\mathrm{Length}\epsilon \leq \mathrm{Length}x$ by
4: $0 \leq \mathrm{Length}x$
7: $\mathrm{Length}\epsilon \leq \mathrm{Length}x \quad \vee \quad \neg\,0 \leq \mathrm{Length}x$

9: $x \preceq x$ by
5: $\mathrm{Length}x \leq \mathrm{Length}x$
2: $x \preceq x \quad \vee \quad \neg\,\mathrm{Length}x \leq \mathrm{Length}x$

10: $\epsilon \preceq x$ by
8: $\mathrm{Length}\epsilon \leq \mathrm{Length}x$
1: $\epsilon \preceq x \quad \vee \quad \neg\,\mathrm{Length}\epsilon \leq \mathrm{Length}x$

11: $\neg\, \epsilon \preceq x$ by

9: $x \preceq x$

0: $\neg\, \epsilon \preceq x \quad \vee \quad \neg\, x \preceq x$

12: QEA by

10: $\epsilon \preceq x$

11: $\neg\, \epsilon \preceq x$