Proof of Theorem 271

The theorem to be proved is

$\mathrm{Q}x \le \mathrm{QS}x$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\,(\mathrm{Q}x) \le (\mathrm{Q}(\mathrm{S}x))]]$

Special cases of the hypothesis and previous results:

0: $\neg\,\mathrm{Q}x \le \mathrm{Q}(\mathrm{S}x)$ from H:x

1: $\mathrm{Q}x \le \mathrm{Q}x$ from 60;Qx

2: $\neg\,\mathrm{S}(\mathrm{R}x) < \mathrm{Q}x \quad \vee \quad \mathrm{Q}(\mathrm{S}x) = \mathrm{Q}x$ from 268;x

3: $\mathrm{R}x < \mathrm{Q}x$ from 166;x

4: $\neg\,\mathrm{S}(\mathrm{R}x) = \mathrm{Q}x \quad \vee \quad 2 \cdot (\mathrm{Q}x) = \mathrm{Q}(\mathrm{S}x)$ from 270;x

5: $(\mathrm{Q}x) + (\mathrm{Q}x) = 2 \cdot (\mathrm{Q}x)$ from 118;Qx

6: $\mathrm{Q}x \le (\mathrm{Q}x) + (\mathrm{Q}x)$ from 71;Qx;Qx

7: $\neg\,\mathrm{R}x < \mathrm{Q}x \quad \vee \quad \mathrm{S}(\mathrm{R}x) \le \mathrm{Q}x$ from 114;Rx;Qx

8: $\mathrm{S}(\mathrm{R}x) < \mathrm{Q}x \quad \vee \quad \neg\,\mathrm{S}(\mathrm{R}x) \le \mathrm{Q}x \quad \vee \quad \mathrm{S}(\mathrm{R}x) = \mathrm{Q}x$ from $56^{\Leftarrow}$;S(Rx);Qx

Equality substitutions:

9: $\neg\,\mathrm{Q}(\mathrm{S}x) = \mathrm{Q}x \quad \vee \quad \mathrm{Q}x \le \mathrm{Q}(\mathrm{S}x) \quad \vee \quad \neg\,\mathrm{Q}x \le \mathrm{Q}x$

10: $\neg\,2 \cdot (\mathrm{Q}x) = \mathrm{Q}(\mathrm{S}x) \quad \vee \quad \neg\,(\mathrm{Q}x) + (\mathrm{Q}x) = 2 \cdot (\mathrm{Q}x) \quad \vee \quad (\mathrm{Q}x) + (\mathrm{Q}x) = \mathrm{Q}(\mathrm{S}x)$

11: $\neg\,(\mathrm{Q}x) + (\mathrm{Q}x) = \mathrm{Q}(\mathrm{S}x) \quad \vee \quad \neg\,\mathrm{Q}x \le (\mathrm{Q}x) + (\mathrm{Q}x) \quad \vee \quad \mathrm{Q}x \le \mathrm{Q}(\mathrm{S}x)$

Inferences:

12: $\neg\,\mathrm{Q}(\mathrm{S}x) = \mathrm{Q}x \quad \vee \quad \neg\,\mathrm{Q}x \le \mathrm{Q}x$ by

0: $\neg\,\mathrm{Q}x \le \mathrm{Q}(\mathrm{S}x)$

9: $\neg\,\mathrm{Q}(\mathrm{S}x) = \mathrm{Q}x \quad \vee \quad \mathrm{Q}x \le \mathrm{Q}(\mathrm{S}x) \quad \vee \quad \neg\,\mathrm{Q}x \le \mathrm{Q}x$

13: $\neg\,(\mathrm{Q}x) + (\mathrm{Q}x) = \mathrm{Q}(\mathrm{S}x) \quad \vee \quad \neg\,\mathrm{Q}x \le (\mathrm{Q}x) + (\mathrm{Q}x)$ by

0: $\neg\,\mathrm{Q}x \le \mathrm{Q}(\mathrm{S}x)$

11: $\neg\,(\mathrm{Q}x) + (\mathrm{Q}x) = \mathrm{Q}(\mathrm{S}x) \quad \vee \quad \neg\,\mathrm{Q}x \le (\mathrm{Q}x) + (\mathrm{Q}x) \quad \vee \quad \mathrm{Q}x \le \mathrm{Q}(\mathrm{S}x)$

14: $\neg\ \mathrm{Q(S}x) = \mathrm{Q}x$ by

1: $\mathrm{Q}x \leq \mathrm{Q}x$

12: $\neg\ \mathrm{Q(S}x) = \mathrm{Q}x \ \lor\ \neg\ \mathrm{Q}x \leq \mathrm{Q}x$

15: $\mathrm{S(R}x) \leq \mathrm{Q}x$ by

3: $\mathrm{R}x < \mathrm{Q}x$

7: $\neg\ \mathrm{R}x < \mathrm{Q}x \ \lor\ \mathrm{S(R}x) \leq \mathrm{Q}x$

16: $\neg\ 2 \cdot (\mathrm{Q}x) = \mathrm{Q(S}x) \ \lor\ (\mathrm{Q}x) + (\mathrm{Q}x) = \mathrm{Q(S}x)$ by

5: $(\mathrm{Q}x) + (\mathrm{Q}x) = 2 \cdot (\mathrm{Q}x)$

10: $\neg\ 2 \cdot (\mathrm{Q}x) = \mathrm{Q(S}x) \ \lor\ \neg\ (\mathrm{Q}x) + (\mathrm{Q}x) = 2 \cdot (\mathrm{Q}x) \ \lor\ (\mathrm{Q}x) + (\mathrm{Q}x) = \mathrm{Q(S}x)$

17: $\neg\ (\mathrm{Q}x) + (\mathrm{Q}x) = \mathrm{Q(S}x)$ by

6: $\mathrm{Q}x \leq (\mathrm{Q}x) + (\mathrm{Q}x)$

13: $\neg\ (\mathrm{Q}x) + (\mathrm{Q}x) = \mathrm{Q(S}x) \ \lor\ \neg\ \mathrm{Q}x \leq (\mathrm{Q}x) + (\mathrm{Q}x)$

18: $\neg\ \mathrm{S(R}x) < \mathrm{Q}x$ by

14: $\neg\ \mathrm{Q(S}x) = \mathrm{Q}x$

2: $\neg\ \mathrm{S(R}x) < \mathrm{Q}x \ \lor\ \mathrm{Q(S}x) = \mathrm{Q}x$

19: $\mathrm{S(R}x) < \mathrm{Q}x \ \lor\ \mathrm{S(R}x) = \mathrm{Q}x$ by

15: $\mathrm{S(R}x) \leq \mathrm{Q}x$

8: $\mathrm{S(R}x) < \mathrm{Q}x \ \lor\ \neg\ \mathrm{S(R}x) \leq \mathrm{Q}x \ \lor\ \mathrm{S(R}x) = \mathrm{Q}x$

20: $\neg\ 2 \cdot (\mathrm{Q}x) = \mathrm{Q(S}x)$ by

17: $\neg\ (\mathrm{Q}x) + (\mathrm{Q}x) = \mathrm{Q(S}x)$

16: $\neg\ 2 \cdot (\mathrm{Q}x) = \mathrm{Q(S}x) \ \lor\ (\mathrm{Q}x) + (\mathrm{Q}x) = \mathrm{Q(S}x)$

21: $\mathrm{S(R}x) = \mathrm{Q}x$ by

18: $\neg\ \mathrm{S(R}x) < \mathrm{Q}x$

19: $\mathrm{S(R}x) < \mathrm{Q}x \ \lor\ \mathrm{S(R}x) = \mathrm{Q}x$

22: $\neg\ \mathrm{S(R}x) = \mathrm{Q}x$ by

20: $\neg\ 2 \cdot (\mathrm{Q}x) = \mathrm{Q(S}x)$

4: $\neg\ \mathrm{S(R}x) = \mathrm{Q}x \ \lor\ 2 \cdot (\mathrm{Q}x) = \mathrm{Q(S}x)$

23: QEA by

21: $\mathrm{S(R}x) = \mathrm{Q}x$

22: $\neg\ \mathrm{S(R}x) = \mathrm{Q}x$