Proof of Theorem 27

The theorem to be proved is

$y - x \neq 0 \quad \rightarrow \quad \mathrm{S}x = y \quad \vee \quad y - \mathrm{S}x \neq 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\,(y - x) = (0)] \quad \& \quad [\neg\,(\mathrm{S}x) = (y)] \quad \& \quad [(y - (\mathrm{S}x)) = (0)]]$

Special cases of the hypothesis and previous results:

0: $\neg\, y - x = 0$ from H:y:x

1: $\neg\,\mathrm{S}x = y$ from H:y:x

2: $y - (\mathrm{S}x) = 0$ from H:y:x

3: $y - x = 0 \quad \vee \quad \neg\, y - (\mathrm{S}x) = 0 \quad \vee \quad y - x = \mathrm{S}0$ from 26;y;x

4: $\neg\, y - x = \mathrm{S}0 \quad \vee \quad \mathrm{S}x = y$ from 24;y;x

Inferences:

5: $\neg\, y - (\mathrm{S}x) = 0 \quad \vee \quad y - x = \mathrm{S}0$ by
0: $\neg\, y - x = 0$
3: $y - x = 0 \quad \vee \quad \neg\, y - (\mathrm{S}x) = 0 \quad \vee \quad y - x = \mathrm{S}0$

6: $\neg\, y - x = \mathrm{S}0$ by
1: $\neg\,\mathrm{S}x = y$
4: $\neg\, y - x = \mathrm{S}0 \quad \vee \quad \mathrm{S}x = y$

7: $y - x = \mathrm{S}0$ by
2: $y - (\mathrm{S}x) = 0$
5: $\neg\, y - (\mathrm{S}x) = 0 \quad \vee \quad y - x = \mathrm{S}0$

8: QEA by
6: $\neg\, y - x = \mathrm{S}0$
7: $y - x = \mathrm{S}0$