Proof of Theorem 268

The theorem to be proved is

$\mathrm{SR}x < \mathrm{Q}x \quad \rightarrow \quad \mathrm{QS}x = \mathrm{Q}x$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(\mathrm{S}(\mathrm{R}x)) < (\mathrm{Q}x)] \quad \& \quad [\neg\, (\mathrm{Q}(\mathrm{S}x)) = (\mathrm{Q}x)]]$

Special cases of the hypothesis and previous results:

0: $\mathrm{S}(\mathrm{R}x) < \mathrm{Q}x$ from H:x

1: $\neg\, \mathrm{Q}(\mathrm{S}x) = \mathrm{Q}x$ from H:x

2: $(\mathrm{Q}x) + (\mathrm{R}x) = \mathrm{S}x$ from 166;x

3: $\mathrm{Q}x$ is a power of two from 166;x

4: $\mathrm{S}((\mathrm{Q}x) + (\mathrm{R}x)) = (\mathrm{Q}x) + (\mathrm{S}(\mathrm{R}x))$ from 12;$\mathrm{Q}x$;$\mathrm{R}x$

5: $\neg\, (\mathrm{Q}x) + (\mathrm{S}(\mathrm{R}x)) = \mathrm{S}(\mathrm{S}x) \quad \vee \quad \neg\, \mathrm{Q}x$ is a power of two $\vee \quad \neg\, \mathrm{S}(\mathrm{R}x) < \mathrm{Q}x \quad \vee \quad \mathrm{Q}(\mathrm{S}x) = \mathrm{Q}x$ from 171;$\mathrm{S}x$;$\mathrm{Q}x$;$\mathrm{S}(\mathrm{R}x)$

Equality substitutions:

6: $\neg\, (\mathrm{Q}x) + (\mathrm{R}x) = \mathrm{S}x \quad \vee \quad \neg\, \mathrm{S}((\mathrm{Q}x) + (\mathrm{R}x)) = (\mathrm{Q}x) + (\mathrm{S}(\mathrm{R}x)) \quad \vee \quad \mathrm{S}(\mathrm{S}x) = (\mathrm{Q}x) + (\mathrm{S}(\mathrm{R}x))$

Inferences:

7: $\neg\, (\mathrm{Q}x) + (\mathrm{S}(\mathrm{R}x)) = \mathrm{S}(\mathrm{S}x) \quad \vee \quad \neg\, \mathrm{Q}x$ is a power of two $\vee \quad \mathrm{Q}(\mathrm{S}x) = \mathrm{Q}x$ by

0: $\mathrm{S}(\mathrm{R}x) < \mathrm{Q}x$

5: $\neg\, (\mathrm{Q}x) + (\mathrm{S}(\mathrm{R}x)) = \mathrm{S}(\mathrm{S}x) \quad \vee \quad \neg\, \mathrm{Q}x$ is a power of two $\vee \quad \neg\, \mathrm{S}(\mathrm{R}x) < \mathrm{Q}x \quad \vee \quad \mathrm{Q}(\mathrm{S}x) = \mathrm{Q}x$

8: $\neg\, (\mathrm{Q}x) + (\mathrm{S}(\mathrm{R}x)) = \mathrm{S}(\mathrm{S}x) \quad \vee \quad \neg\, \mathrm{Q}x$ is a power of two by

1: $\neg\, \mathrm{Q}(\mathrm{S}x) = \mathrm{Q}x$

7: $\neg\, (\mathrm{Q}x) + (\mathrm{S}(\mathrm{R}x)) = \mathrm{S}(\mathrm{S}x) \quad \vee \quad \neg\, \mathrm{Q}x$ is a power of two $\vee \quad \mathrm{Q}(\mathrm{S}x) = \mathrm{Q}x$

9: $\neg\, \mathrm{S}((\mathrm{Q}x) + (\mathrm{R}x)) = (\mathrm{Q}x) + (\mathrm{S}(\mathrm{R}x)) \quad \vee \quad (\mathrm{Q}x) + (\mathrm{S}(\mathrm{R}x)) = \mathrm{S}(\mathrm{S}x)$ by

2: $(\mathrm{Q}x) + (\mathrm{R}x) = \mathrm{S}x$

6: $\neg\, (\mathrm{Q}x) + (\mathrm{R}x) = \mathrm{S}x \quad \vee \quad \neg\, \mathrm{S}((\mathrm{Q}x) + (\mathrm{R}x)) = (\mathrm{Q}x) + (\mathrm{S}(\mathrm{R}x)) \quad \vee \quad (\mathrm{Q}x) + (\mathrm{S}(\mathrm{R}x)) = \mathrm{S}(\mathrm{S}x)$

10: $\neg\,(\mathrm{Q}x) + (\mathrm{S}(\mathrm{R}x)) = \mathrm{S}(\mathrm{S}x)$ by

3: $\mathrm{Q}x$ is a power of two

8: $\neg\,(\mathrm{Q}x) + (\mathrm{S}(\mathrm{R}x)) = \mathrm{S}(\mathrm{S}x) \quad \vee \quad \neg\,\mathrm{Q}x$ is a power of two

11: $(\mathrm{Q}x) + (\mathrm{S}(\mathrm{R}x)) = \mathrm{S}(\mathrm{S}x)$ by

4: $\mathrm{S}((\mathrm{Q}x) + (\mathrm{R}x)) = (\mathrm{Q}x) + (\mathrm{S}(\mathrm{R}x))$

9: $\neg\,\mathrm{S}((\mathrm{Q}x) + (\mathrm{R}x)) = (\mathrm{Q}x) + (\mathrm{S}(\mathrm{R}x)) \quad \vee \quad (\mathrm{Q}x) + (\mathrm{S}(\mathrm{R}x)) = \mathrm{S}(\mathrm{S}x)$

12: QEA by

10: $\neg\,(\mathrm{Q}x) + (\mathrm{S}(\mathrm{R}x)) = \mathrm{S}(\mathrm{S}x)$

11: $(\mathrm{Q}x) + (\mathrm{S}(\mathrm{R}x)) = \mathrm{S}(\mathrm{S}x)$