Proof of Theorem 267

The theorem to be proved is

$x \preceq y \quad \rightarrow \quad x \leq 2 \cdot \mathrm{Q}y$ ⋆

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x) \preceq (y)] \quad \& \quad [\neg\,(x) \leq (2 \cdot (\mathrm{Q}y))]]$

Special cases of the hypothesis and previous results:

0: $x \preceq y$ from H:x:y

1: $\neg\, x \leq 2 \cdot (\mathrm{Q}y)$ from H:x:y

2: $\neg\, x \preceq y \;\vee\; \mathrm{Length}x \leq \mathrm{Length}y$ from 264$^{\rightarrow}$;x;y

3: $\neg\, \mathrm{Length}x \leq \mathrm{Length}y \;\vee\; 2{\uparrow}(\mathrm{Length}x) \leq 2{\uparrow}(\mathrm{Length}y)$ from 263;Length$x$;Length$y$

4: $2 \uparrow (\mathrm{Length}x) = \mathrm{Q}x$ from 261;x

5: $2 \uparrow (\mathrm{Length}y) = \mathrm{Q}y$ from 261;y

6: $\mathrm{S}x < 2 \cdot (\mathrm{Q}x)$ from 158;x

7: $x \leq \mathrm{S}x$ from 63;x

8: $\neg\, \mathrm{S}x < 2 \cdot (\mathrm{Q}x) \;\vee\; \mathrm{S}x \leq 2 \cdot (\mathrm{Q}x)$ from 56$^{\rightarrow}$;Sx;2 · (Qx)

9: $\neg\, x \leq \mathrm{S}x \;\vee\; \neg\, \mathrm{S}x \leq 2 \cdot (\mathrm{Q}x) \;\vee\; x \leq 2 \cdot (\mathrm{Q}x)$ from 73;x;Sx;2 · (Qx)

10: $\neg\, \mathrm{Q}x \leq \mathrm{Q}y \;\vee\; 2 \cdot (\mathrm{Q}x) \leq 2 \cdot (\mathrm{Q}y)$ from 266;Qx;Qy

11: $\neg\, x \leq 2{\cdot}(\mathrm{Q}x) \;\vee\; \neg\, 2{\cdot}(\mathrm{Q}x) \leq 2{\cdot}(\mathrm{Q}y) \;\vee\; x \leq 2{\cdot}(\mathrm{Q}y)$ from 73;x;2·(Qx);2·(Qy)

Equality substitutions:

12: $\neg\, 2 \uparrow (\mathrm{Length}x) = \mathrm{Q}x \;\vee\; \neg\, 2 \uparrow (\mathrm{Length}x) \leq 2 \uparrow (\mathrm{Length}y) \;\vee\; \mathrm{Q}x \leq 2 \uparrow (\mathrm{Length}y)$

13: $\neg\, 2 \uparrow (\mathrm{Length}y) = \mathrm{Q}y \;\vee\; \neg\, \mathrm{Q}x \leq 2 \uparrow (\mathrm{Length}y) \;\vee\; \mathrm{Q}x \leq \mathrm{Q}y$

Inferences:

14: $\mathrm{Length}x \leq \mathrm{Length}y$ by

0: $x \preceq y$

2: $\neg\, x \preceq y \;\vee\; \mathrm{Length}x \leq \mathrm{Length}y$

15: $\neg\, x \leq 2 \cdot (\mathrm{Q}x) \quad \vee \quad \neg\, 2 \cdot (\mathrm{Q}x) \leq 2 \cdot (\mathrm{Q}y)$ by

1: $\neg\, x \leq 2 \cdot (\mathrm{Q}y)$

11: $\neg\, x \leq 2 \cdot (\mathrm{Q}x) \quad \vee \quad \neg\, 2 \cdot (\mathrm{Q}x) \leq 2 \cdot (\mathrm{Q}y) \quad \vee \quad x \leq 2 \cdot (\mathrm{Q}y)$

16: $\neg\, 2 \uparrow (\mathrm{Length}x) \leq 2 \uparrow (\mathrm{Length}y) \quad \vee \quad \mathrm{Q}x \leq 2 \uparrow (\mathrm{Length}y)$ by

4: $2 \uparrow (\mathrm{Length}x) = \mathrm{Q}x$

12: $\neg\, 2 \uparrow (\mathrm{Length}x) = \mathrm{Q}x \quad \vee \quad \neg\, 2 \uparrow (\mathrm{Length}x) \leq 2 \uparrow (\mathrm{Length}y) \quad \vee \quad \mathrm{Q}x \leq 2 \uparrow (\mathrm{Length}y)$

17: $\neg\, \mathrm{Q}x \leq 2 \uparrow (\mathrm{Length}y) \quad \vee \quad \mathrm{Q}x \leq \mathrm{Q}y$ by

5: $2 \uparrow (\mathrm{Length}y) = \mathrm{Q}y$

13: $\neg\, 2 \uparrow (\mathrm{Length}y) = \mathrm{Q}y \quad \vee \quad \neg\, \mathrm{Q}x \leq 2 \uparrow (\mathrm{Length}y) \quad \vee \quad \mathrm{Q}x \leq \mathrm{Q}y$

18: $\mathrm{S}x \leq 2 \cdot (\mathrm{Q}x)$ by

6: $\mathrm{S}x < 2 \cdot (\mathrm{Q}x)$

8: $\neg\, \mathrm{S}x < 2 \cdot (\mathrm{Q}x) \quad \vee \quad \mathrm{S}x \leq 2 \cdot (\mathrm{Q}x)$

19: $\neg\, \mathrm{S}x \leq 2 \cdot (\mathrm{Q}x) \quad \vee \quad x \leq 2 \cdot (\mathrm{Q}x)$ by

7: $x \leq \mathrm{S}x$

9: $\neg\, x \leq \mathrm{S}x \quad \vee \quad \neg\, \mathrm{S}x \leq 2 \cdot (\mathrm{Q}x) \quad \vee \quad x \leq 2 \cdot (\mathrm{Q}x)$

20: $2 \uparrow (\mathrm{Length}x) \leq 2 \uparrow (\mathrm{Length}y)$ by

14: $\mathrm{Length}x \leq \mathrm{Length}y$

3: $\neg\, \mathrm{Length}x \leq \mathrm{Length}y \quad \vee \quad 2 \uparrow (\mathrm{Length}x) \leq 2 \uparrow (\mathrm{Length}y)$

21: $x \leq 2 \cdot (\mathrm{Q}x)$ by

18: $\mathrm{S}x \leq 2 \cdot (\mathrm{Q}x)$

19: $\neg\, \mathrm{S}x \leq 2 \cdot (\mathrm{Q}x) \quad \vee \quad x \leq 2 \cdot (\mathrm{Q}x)$

22: $\mathrm{Q}x \leq 2 \uparrow (\mathrm{Length}y)$ by

20: $2 \uparrow (\mathrm{Length}x) \leq 2 \uparrow (\mathrm{Length}y)$

16: $\neg\, 2 \uparrow (\mathrm{Length}x) \leq 2 \uparrow (\mathrm{Length}y) \quad \vee \quad \mathrm{Q}x \leq 2 \uparrow (\mathrm{Length}y)$

23: $\neg\, 2 \cdot (\mathrm{Q}x) \leq 2 \cdot (\mathrm{Q}y)$ by

21: $x \leq 2 \cdot (\mathrm{Q}x)$

15: $\neg\, x \leq 2 \cdot (\mathrm{Q}x) \quad \vee \quad \neg\, 2 \cdot (\mathrm{Q}x) \leq 2 \cdot (\mathrm{Q}y)$

24: $\mathrm{Q}x \leq \mathrm{Q}y$ by

22: $\mathrm{Q}x \leq 2 \uparrow (\mathrm{Length}y)$

17: $\neg\, \mathrm{Q}x \leq 2 \uparrow (\mathrm{Length}y) \quad \vee \quad \mathrm{Q}x \leq \mathrm{Q}y$

25: $\neg\, \mathrm{Q}x \leq \mathrm{Q}y$ by

23: $\neg\, 2 \cdot (\mathrm{Q}x) \leq 2 \cdot (\mathrm{Q}y)$

10: $\neg\, \mathrm{Q}x \leq \mathrm{Q}y \quad \vee \quad 2 \cdot (\mathrm{Q}x) \leq 2 \cdot (\mathrm{Q}y)$

26: QEA by

24: $\mathrm{Q}x \leq \mathrm{Q}y$

25: $\neg\ \mathrm{Q}x \leq \mathrm{Q}y$