Proof of Theorem 263b

The theorem to be proved is

$x \le 0 \quad \rightarrow \quad 2 \uparrow x \le 2 \uparrow 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x) \le (0)] \quad \& \quad [\neg\, (2 \uparrow x) \le (2 \uparrow 0)]]$

Special cases of the hypothesis and previous results:

0: $x \le 0$ from H:x

1: $\neg\, 2 \uparrow x \le 2 \uparrow 0$ from H:x

2: $\neg\, x \le 0 \quad \vee \quad 0 = x$ from 57;x

3: $2 \uparrow 0 \le 2 \uparrow 0$ from 60;$2 \uparrow 0$

Equality substitutions:

4: $\neg\; x = 0 \quad \vee \quad 2 \uparrow (x) \le 2 \uparrow 0 \quad \vee \quad \neg\; 2 \uparrow (0) \le 2 \uparrow 0$

Inferences:

5: $0 = x$ by
0: $x \le 0$
2: $\neg\, x \le 0 \quad \vee \quad 0 = x$

6: $\neg\, 0 = x \quad \vee \quad \neg\, 2 \uparrow 0 \le 2 \uparrow 0$ by
1: $\neg\, 2 \uparrow x \le 2 \uparrow 0$
4: $\neg\, 0 = x \quad \vee \quad 2 \uparrow x \le 2 \uparrow 0 \quad \vee \quad \neg\, 2 \uparrow 0 \le 2 \uparrow 0$

7: $\neg\, 0 = x$ by
3: $2 \uparrow 0 \le 2 \uparrow 0$
6: $\neg\, 0 = x \quad \vee \quad \neg\, 2 \uparrow 0 \le 2 \uparrow 0$

8: QEA by
5: $0 = x$
7: $\neg\, 0 = x$