Proof of Theorem 261b

The theorem to be proved is

$\mathrm{Q}\epsilon = 2 \uparrow (\mathrm{Length}\,\epsilon)$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\,(\mathrm{Q}\epsilon) = (2 \uparrow (\mathrm{Length}\epsilon))]]$

Special cases of the hypothesis and previous results:

0: $\neg\, 2 \uparrow (\mathrm{Length}\epsilon) = \mathrm{Q}\epsilon$ from H

1: $\mathrm{Length}\epsilon = 0$ from 259

2: $2 \uparrow 0 = 1$ from 126;2

3: $\mathrm{Q}\epsilon = 1$ from 189

Equality substitutions:

4: $\neg\, \mathrm{Length}\epsilon = 0 \quad \vee \quad 2 \uparrow (\mathrm{Length}\epsilon) = \mathrm{Q}\epsilon \quad \vee \quad \neg\, 2 \uparrow (0) = \mathrm{Q}\epsilon$

5: $\neg\, 2 \uparrow 0 = 1 \quad \vee \quad 2 \uparrow 0 = \mathrm{Q}\epsilon \quad \vee \quad \neg\, 1 = \mathrm{Q}\epsilon$

Inferences:

6: $\neg\, \mathrm{Length}\epsilon = 0 \quad \vee \quad \neg\, 2 \uparrow 0 = \mathrm{Q}\epsilon$ by
0: $\neg\, 2 \uparrow (\mathrm{Length}\epsilon) = \mathrm{Q}\epsilon$
4: $\neg\, \mathrm{Length}\epsilon = 0 \quad \vee \quad 2 \uparrow (\mathrm{Length}\epsilon) = \mathrm{Q}\epsilon \quad \vee \quad \neg\, 2 \uparrow 0 = \mathrm{Q}\epsilon$

7: $\neg\, 2 \uparrow 0 = \mathrm{Q}\epsilon$ by
1: $\mathrm{Length}\epsilon = 0$
6: $\neg\, \mathrm{Length}\epsilon = 0 \quad \vee \quad \neg\, 2 \uparrow 0 = \mathrm{Q}\epsilon$

8: $2 \uparrow 0 = \mathrm{Q}\epsilon \quad \vee \quad \neg\, \mathrm{Q}\epsilon = 1$ by
2: $2 \uparrow 0 = 1$
5: $\neg\, 2 \uparrow 0 = 1 \quad \vee \quad 2 \uparrow 0 = \mathrm{Q}\epsilon \quad \vee \quad \neg\, \mathrm{Q}\epsilon = 1$

9: $2 \uparrow 0 = \mathrm{Q}\epsilon$ by
3: $\mathrm{Q}\epsilon = 1$
8: $2 \uparrow 0 = \mathrm{Q}\epsilon \quad \vee \quad \neg\, \mathrm{Q}\epsilon = 1$

10: QEA by
7: $\neg\, 2 \uparrow 0 = \mathrm{Q}\epsilon$
9: $2 \uparrow 0 = \mathrm{Q}\epsilon$