Proof of Theorem 260b

The theorem to be proved is

$\mathrm{Length}(x \oplus \epsilon) = \mathrm{Length}\, x + \mathrm{Length}\, \epsilon$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\ (\mathrm{Length}(x \oplus \epsilon)) = ((\mathrm{Length} x) + (\mathrm{Length} \epsilon))]]$

Special cases of the hypothesis and previous results:

0: $\neg\ (\mathrm{Length} x) + (\mathrm{Length} \epsilon) = \mathrm{Length}(x \oplus \epsilon)$ from H:x

1: $x \oplus \epsilon = x$ from 196;x

2: $\mathrm{Length} \epsilon = 0$ from 259

3: $(\mathrm{Length} x) + 0 = \mathrm{Length} x$ from 12;$\mathrm{Length} x$

Equality substitutions:

4: $\neg\ x \oplus \epsilon = x \ \vee\ (\mathrm{Length} x) + (\mathrm{Length} \epsilon) = \mathrm{Length}(x \oplus \epsilon) \ \vee\ \neg\ (\mathrm{Length} x) + (\mathrm{Length} \epsilon) = \mathrm{Length}(x)$

5: $\neg\ \mathrm{Length} \epsilon = 0 \ \vee\ (\mathrm{Length} x) + (\mathrm{Length} \epsilon) = \mathrm{Length} x \ \vee\ \neg\ (\mathrm{Length} x) + (0) = \mathrm{Length} x$

Inferences:

6: $\neg\ x \oplus \epsilon = x \ \vee\ \neg\ (\mathrm{Length} x) + (\mathrm{Length} \epsilon) = \mathrm{Length} x$ by

0: $\neg\ (\mathrm{Length} x) + (\mathrm{Length} \epsilon) = \mathrm{Length}(x \oplus \epsilon)$

4: $\neg\ x \oplus \epsilon = x \ \vee\ (\mathrm{Length} x) + (\mathrm{Length} \epsilon) = \mathrm{Length}(x \oplus \epsilon) \ \vee\ \neg\ (\mathrm{Length} x) + (\mathrm{Length} \epsilon) = \mathrm{Length} x$

7: $\neg\ (\mathrm{Length} x) + (\mathrm{Length} \epsilon) = \mathrm{Length} x$ by

1: $x \oplus \epsilon = x$

6: $\neg\ x \oplus \epsilon = x \ \vee\ \neg\ (\mathrm{Length} x) + (\mathrm{Length} \epsilon) = \mathrm{Length} x$

8: $(\mathrm{Length} x) + (\mathrm{Length} \epsilon) = \mathrm{Length} x \ \vee\ \neg\ (\mathrm{Length} x) + 0 = \mathrm{Length} x$ by

2: $\mathrm{Length} \epsilon = 0$

5: $\neg\ \mathrm{Length} \epsilon = 0 \ \vee\ (\mathrm{Length} x) + (\mathrm{Length} \epsilon) = \mathrm{Length} x \ \vee\ \neg\ (\mathrm{Length} x) + 0 = \mathrm{Length} x$

9: $(\mathrm{Length} x) + (\mathrm{Length} \epsilon) = \mathrm{Length} x$ by

3: $(\mathrm{Length} x) + 0 = \mathrm{Length} x$

8: $(\mathrm{Length} x) + (\mathrm{Length} \epsilon) = \mathrm{Length} x \quad \vee \quad \neg\, (\mathrm{Length} x) + 0 = \mathrm{Length} x$

10: QEA by

7: $\neg\, (\mathrm{Length} x) + (\mathrm{Length} \epsilon) = \mathrm{Length} x$

9: $(\mathrm{Length} x) + (\mathrm{Length} \epsilon) = \mathrm{Length} x$