Proof of Theorem 26

The theorem to be proved is

$y - x \neq 0 \quad \& \quad y - \mathrm{S}x = 0 \quad \rightarrow \quad y - x = \mathrm{S}0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\, (y - x) = (0)] \quad \& \quad [(y - (\mathrm{S}x)) = (0)] \quad \& \quad [\neg\, (y - x) = (\mathrm{S}0)]]$

Special cases of the hypothesis and previous results:

0: $\neg\, y - x = 0$ from H:y:x

1: $y - (\mathrm{S}x) = 0$ from H:y:x

2: $\neg\, y - x = \mathrm{S}0$ from H:y:x

3: $\mathrm{P}(y - x) = y - (\mathrm{S}x)$ from 17;y;x

4: $y - x = 0 \quad \vee \quad \neg\, \mathrm{P}(y - x) = 0 \quad \vee \quad y - x = \mathrm{S}0$ from 25;$y - x$

Equality substitutions:

5: $\neg\, y - (\mathrm{S}x) = 0 \quad \vee \quad \neg\, \mathrm{P}(y - x) = y - (\mathrm{S}x) \quad \vee \quad \mathrm{P}(y - x) = 0$

Inferences:

6: $\neg\, \mathrm{P}(y - x) = 0 \quad \vee \quad y - x = \mathrm{S}0$ by
- 0: $\neg\, y - x = 0$
- 4: $y - x = 0 \quad \vee \quad \neg\, \mathrm{P}(y - x) = 0 \quad \vee \quad y - x = \mathrm{S}0$

7: $\neg\, \mathrm{P}(y - x) = y - (\mathrm{S}x) \quad \vee \quad \mathrm{P}(y - x) = 0$ by
- 1: $y - (\mathrm{S}x) = 0$
- 5: $\neg\, y - (\mathrm{S}x) = 0 \quad \vee \quad \neg\, \mathrm{P}(y - x) = y - (\mathrm{S}x) \quad \vee \quad \mathrm{P}(y - x) = 0$

8: $\neg\, \mathrm{P}(y - x) = 0$ by
- 2: $\neg\, y - x = \mathrm{S}0$
- 6: $\neg\, \mathrm{P}(y - x) = 0 \quad \vee \quad y - x = \mathrm{S}0$

9: $\mathrm{P}(y - x) = 0$ by
- 3: $\mathrm{P}(y - x) = y - (\mathrm{S}x)$
- 7: $\neg\, \mathrm{P}(y - x) = y - (\mathrm{S}x) \quad \vee \quad \mathrm{P}(y - x) = 0$

10: QEA by
- 8: $\neg\, \mathrm{P}(y - x) = 0$
- 9: $\mathrm{P}(y - x) = 0$