Proof of Theorem 258

The theorem to be proved is

$x \neq \epsilon \quad \rightarrow \quad \text{Chop}\, x < x$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\,(x) = (\epsilon)] \quad \& \quad [\neg\,(\text{Chop}x) < (x)]]$

Special cases of the hypothesis and previous results:

0: $\neg\,\epsilon = x$ from H:x

1: $\neg\,\text{Chop}x < x$ from H:x

2: $\epsilon = x \;\vee\; (\text{Chop}x) \oplus \underline{0} = x \;\vee\; (\text{Chop}x) \oplus \underline{1} = x$ from 253;x

3: $\text{Chop}x < (\text{Chop}x) \oplus \underline{0}$ from 255;Chop$x$

4: $\text{Chop}x < (\text{Chop}x) \oplus \underline{1}$ from 257;Chop$x$

Equality substitutions:

5: $\neg\,(\text{Chop}x) \oplus \underline{0} = x \;\vee\; \neg\,\text{Chop}x < (\text{Chop}x) \oplus \underline{0} \;\vee\; \text{Chop}x < x$

6: $\neg\,(\text{Chop}x) \oplus \underline{1} = x \;\vee\; \neg\,\text{Chop}x < (\text{Chop}x) \oplus \underline{1} \;\vee\; \text{Chop}x < x$

Inferences:

7: $(\text{Chop}x) \oplus \underline{0} = x \;\vee\; (\text{Chop}x) \oplus \underline{1} = x$ by

0: $\neg\,\epsilon = x$

2: $\epsilon = x \;\vee\; (\text{Chop}x) \oplus \underline{0} = x \;\vee\; (\text{Chop}x) \oplus \underline{1} = x$

8: $\neg\,(\text{Chop}x) \oplus \underline{0} = x \;\vee\; \neg\,\text{Chop}x < (\text{Chop}x) \oplus \underline{0}$ by

1: $\neg\,\text{Chop}x < x$

5: $\neg\,(\text{Chop}x) \oplus \underline{0} = x \;\vee\; \neg\,\text{Chop}x < (\text{Chop}x) \oplus \underline{0} \;\vee\; \text{Chop}x < x$

9: $\neg\,(\text{Chop}x) \oplus \underline{1} = x \;\vee\; \neg\,\text{Chop}x < (\text{Chop}x) \oplus \underline{1}$ by

1: $\neg\,\text{Chop}x < x$

6: $\neg\,(\text{Chop}x) \oplus \underline{1} = x \;\vee\; \neg\,\text{Chop}x < (\text{Chop}x) \oplus \underline{1} \;\vee\; \text{Chop}x < x$

10: $\neg\,(\text{Chop}x) \oplus \underline{0} = x$ by

3: $\text{Chop}x < (\text{Chop}x) \oplus \underline{0}$

8: $\neg\,(\text{Chop}x) \oplus \underline{0} = x \;\vee\; \neg\,\text{Chop}x < (\text{Chop}x) \oplus \underline{0}$

11: $\neg\,(\mathrm{Chop}x) \oplus \underline{1} = x$ by

4: $\mathrm{Chop}x < (\mathrm{Chop}x) \oplus \underline{1}$

9: $\neg\,(\mathrm{Chop}x) \oplus \underline{1} = x \quad \vee \quad \neg\,\mathrm{Chop}x < (\mathrm{Chop}x) \oplus \underline{1}$

12: $(\mathrm{Chop}x) \oplus \underline{1} = x$ by

10: $\neg\,(\mathrm{Chop}x) \oplus \underline{0} = x$

7: $(\mathrm{Chop}x) \oplus \underline{0} = x \quad \vee \quad (\mathrm{Chop}x) \oplus \underline{1} = x$

13: QEA by

11: $\neg\,(\mathrm{Chop}x) \oplus \underline{1} = x$

12: $(\mathrm{Chop}x) \oplus \underline{1} = x$