Proof of Theorem 256

The theorem to be proved is

$x + 1 = \mathrm{S}x$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\ (x + 1) = (\mathrm{S}x)]]$

Special cases of the hypothesis and previous results:

0: $\neg\ x + 1 = \mathrm{S}x$ from H:x

1: $\mathrm{S}0 = 1$ from 115

2: $x + 0 = x$ from 12;x;0

3: $\mathrm{S}(x + 0) = x + (\mathrm{S}0)$ from 12;x;0

Equality substitutions:

4: $\neg\ \mathrm{S}0 = 1 \quad \vee \quad \neg\ x + (\mathrm{S}0) = \mathrm{S}x \quad \vee \quad x + (1) = \mathrm{S}x$

5: $\neg\ x + 0 = x \quad \vee \quad \neg\ \mathrm{S}(x + 0) = x + (\mathrm{S}0) \quad \vee \quad \mathrm{S}(x) = x + (\mathrm{S}0)$

Inferences:

6: $\neg\ \mathrm{S}0 = 1 \quad \vee \quad \neg\ x + (\mathrm{S}0) = \mathrm{S}x$ by

0: $\neg\ x + 1 = \mathrm{S}x$

4: $\neg\ \mathrm{S}0 = 1 \quad \vee \quad \neg\ x + (\mathrm{S}0) = \mathrm{S}x \quad \vee \quad x + 1 = \mathrm{S}x$

7: $\neg\ x + (\mathrm{S}0) = \mathrm{S}x$ by

1: $\mathrm{S}0 = 1$

6: $\neg\ \mathrm{S}0 = 1 \quad \vee \quad \neg\ x + (\mathrm{S}0) = \mathrm{S}x$

8: $\neg\ \mathrm{S}(x + 0) = x + (\mathrm{S}0) \quad \vee \quad x + (\mathrm{S}0) = \mathrm{S}x$ by

2: $x + 0 = x$

5: $\neg\ x + 0 = x \quad \vee \quad \neg\ \mathrm{S}(x + 0) = x + (\mathrm{S}0) \quad \vee \quad x + (\mathrm{S}0) = \mathrm{S}x$

9: $x + (\mathrm{S}0) = \mathrm{S}x$ by

3: $\mathrm{S}(x + 0) = x + (\mathrm{S}0)$

8: $\neg\ \mathrm{S}(x + 0) = x + (\mathrm{S}0) \quad \vee \quad x + (\mathrm{S}0) = \mathrm{S}x$

10: QEA by

7: $\neg\ x + (\mathrm{S}0) = \mathrm{S}x$

9: $x + (\mathrm{S}0) = \mathrm{S}x$