Proof of Theorem 254

The theorem to be proved is

$y \neq \epsilon \quad \to \quad \mathrm{Chop}(x \oplus y) = x \oplus \mathrm{Chop}\, y$ ⋆

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\, (y) = (\epsilon)] \quad \& \quad [\neg\, (\mathrm{Chop}(x \oplus y)) = (x \oplus (\mathrm{Chop} y))]]$

Special cases of the hypothesis and previous results:

0: $\neg\, \epsilon = y$ from H:y:x

1: $\neg\, \mathrm{Chop}(x \oplus y) = x \oplus (\mathrm{Chop} y)$ from H:y:x

2: $\epsilon = y \quad \vee \quad (\mathrm{Chop} y) \oplus \underline{0} = y \quad \vee \quad (\mathrm{Chop} y) \oplus \underline{1} = y$ from 253;y

3: $x \oplus ((\mathrm{Chop} y) \oplus \underline{0}) = (x \oplus (\mathrm{Chop} y)) \oplus \underline{0}$ from 183;x;Chop$y$;$\underline{0}$

4: $x \oplus ((\mathrm{Chop} y) \oplus \underline{1}) = (x \oplus (\mathrm{Chop} y)) \oplus \underline{1}$ from 183;x;Chop$y$;$\underline{1}$

5: $\mathrm{Chop}((x \oplus (\mathrm{Chop} y)) \oplus \underline{0}) = x \oplus (\mathrm{Chop} y)$ from 243;$x \oplus (\mathrm{Chop} y)$

6: $\mathrm{Chop}((x \oplus (\mathrm{Chop} y)) \oplus \underline{1}) = x \oplus (\mathrm{Chop} y)$ from 244;$x \oplus (\mathrm{Chop} y)$

Equality substitutions:

7: $\neg\, (\mathrm{Chop} y) \oplus \underline{0} = y \quad \vee \quad \neg\, x \oplus ((\mathrm{Chop} y) \oplus \underline{0}) = (x \oplus (\mathrm{Chop} y)) \oplus \underline{0} \quad \vee \quad x \oplus (y) = (x \oplus (\mathrm{Chop} y)) \oplus \underline{0}$

8: $\neg\, (\mathrm{Chop} y) \oplus \underline{1} = y \quad \vee \quad \neg\, x \oplus ((\mathrm{Chop} y) \oplus \underline{1}) = (x \oplus (\mathrm{Chop} y)) \oplus \underline{1} \quad \vee \quad x \oplus (y) = (x \oplus (\mathrm{Chop} y)) \oplus \underline{1}$

9: $\neg\, (x \oplus (\mathrm{Chop} y)) \oplus \underline{0} = x \oplus y \quad \vee \quad \neg\, \mathrm{Chop}((x \oplus (\mathrm{Chop} y)) \oplus \underline{0}) = x \oplus (\mathrm{Chop} y) \quad \vee \quad \mathrm{Chop}(x \oplus y) = x \oplus (\mathrm{Chop} y)$

10: $\neg\, (x \oplus (\mathrm{Chop} y)) \oplus \underline{1} = x \oplus y \quad \vee \quad \neg\, \mathrm{Chop}((x \oplus (\mathrm{Chop} y)) \oplus \underline{1}) = x \oplus (\mathrm{Chop} y) \quad \vee \quad \mathrm{Chop}(x \oplus y) = x \oplus (\mathrm{Chop} y)$

Inferences:

11: $(\mathrm{Chop} y) \oplus \underline{0} = y \quad \vee \quad (\mathrm{Chop} y) \oplus \underline{1} = y$ by

0: $\neg\, \epsilon = y$

2: $\epsilon = y \quad \vee \quad (\mathrm{Chop} y) \oplus \underline{0} = y \quad \vee \quad (\mathrm{Chop} y) \oplus \underline{1} = y$

12: $\neg\,(x \oplus (\text{Chop}y)) \oplus \underline{0} = x \oplus y \quad \vee \quad \neg\,\text{Chop}((x \oplus (\text{Chop}y)) \oplus \underline{0}) = x \oplus (\text{Chop}y)$ by

1: $\neg\,\text{Chop}(x \oplus y) = x \oplus (\text{Chop}y)$

9: $\neg\,(x \oplus (\text{Chop}y)) \oplus \underline{0} = x \oplus y \quad \vee \quad \neg\,\text{Chop}((x \oplus (\text{Chop}y)) \oplus \underline{0}) = x \oplus (\text{Chop}y) \quad \vee \quad \text{Chop}(x \oplus y) = x \oplus (\text{Chop}y)$

13: $\neg\,(x \oplus (\text{Chop}y)) \oplus \underline{1} = x \oplus y \quad \vee \quad \neg\,\text{Chop}((x \oplus (\text{Chop}y)) \oplus \underline{1}) = x \oplus (\text{Chop}y)$ by

1: $\neg\,\text{Chop}(x \oplus y) = x \oplus (\text{Chop}y)$

10: $\neg\,(x \oplus (\text{Chop}y)) \oplus \underline{1} = x \oplus y \quad \vee \quad \neg\,\text{Chop}((x \oplus (\text{Chop}y)) \oplus \underline{1}) = x \oplus (\text{Chop}y) \quad \vee \quad \text{Chop}(x \oplus y) = x \oplus (\text{Chop}y)$

14: $\neg\,(\text{Chop}y) \oplus \underline{0} = y \quad \vee \quad (x \oplus (\text{Chop}y)) \oplus \underline{0} = x \oplus y$ by

3: $x \oplus ((\text{Chop}y) \oplus \underline{0}) = (x \oplus (\text{Chop}y)) \oplus \underline{0}$

7: $\neg\,(\text{Chop}y) \oplus \underline{0} = y \quad \vee \quad \neg\,x \oplus ((\text{Chop}y) \oplus \underline{0}) = (x \oplus (\text{Chop}y)) \oplus \underline{0} \quad \vee \quad (x \oplus (\text{Chop}y)) \oplus \underline{0} = x \oplus y$

15: $\neg\,(\text{Chop}y) \oplus \underline{1} = y \quad \vee \quad (x \oplus (\text{Chop}y)) \oplus \underline{1} = x \oplus y$ by

4: $x \oplus ((\text{Chop}y) \oplus \underline{1}) = (x \oplus (\text{Chop}y)) \oplus \underline{1}$

8: $\neg\,(\text{Chop}y) \oplus \underline{1} = y \quad \vee \quad \neg\,x \oplus ((\text{Chop}y) \oplus \underline{1}) = (x \oplus (\text{Chop}y)) \oplus \underline{1} \quad \vee \quad (x \oplus (\text{Chop}y)) \oplus \underline{1} = x \oplus y$

16: $\neg\,(x \oplus (\text{Chop}y)) \oplus \underline{0} = x \oplus y$ by

5: $\text{Chop}((x \oplus (\text{Chop}y)) \oplus \underline{0}) = x \oplus (\text{Chop}y)$

12: $\neg\,(x \oplus (\text{Chop}y)) \oplus \underline{0} = x \oplus y \quad \vee \quad \neg\,\text{Chop}((x \oplus (\text{Chop}y)) \oplus \underline{0}) = x \oplus (\text{Chop}y)$

17: $\neg\,(x \oplus (\text{Chop}y)) \oplus \underline{1} = x \oplus y$ by

6: $\text{Chop}((x \oplus (\text{Chop}y)) \oplus \underline{1}) = x \oplus (\text{Chop}y)$

13: $\neg\,(x \oplus (\text{Chop}y)) \oplus \underline{1} = x \oplus y \quad \vee \quad \neg\,\text{Chop}((x \oplus (\text{Chop}y)) \oplus \underline{1}) = x \oplus (\text{Chop}y)$

18: $\neg\,(\text{Chop}y) \oplus \underline{0} = y$ by

16: $\neg\,(x \oplus (\text{Chop}y)) \oplus \underline{0} = x \oplus y$

14: $\neg\,(\text{Chop}y) \oplus \underline{0} = y \quad \vee \quad (x \oplus (\text{Chop}y)) \oplus \underline{0} = x \oplus y$

19: $\neg\,(\text{Chop}y) \oplus \underline{1} = y$ by

17: $\neg\,(x \oplus (\text{Chop}y)) \oplus \underline{1} = x \oplus y$

15: $\neg\,(\text{Chop}y) \oplus \underline{1} = y \quad \vee \quad (x \oplus (\text{Chop}y)) \oplus \underline{1} = x \oplus y$

20: $(\text{Chop}y) \oplus \underline{1} = y$ by

18: $\neg\,(\text{Chop}y) \oplus \underline{0} = y$

11: $(\text{Chop}y) \oplus \underline{0} = y \quad \vee \quad (\text{Chop}y) \oplus \underline{1} = y$

21: QEA by

$\neg\,(\mathrm{Chop}\, y) \oplus \underline{1} = y$

$(\mathrm{Chop}\, y) \oplus \underline{1} = y$