Proof of Theorem 253

The theorem to be proved is

$x \neq \epsilon \quad \rightarrow \quad x = \text{Chop}\, x \oplus \underline{0} \quad \vee \quad x = \text{Chop}\, x \oplus \underline{1}$ ⋆⋆

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\,(x) = (\epsilon)] \quad \& \quad [\neg\,(x) = ((\text{Chop}x) \oplus \underline{0})] \quad \& \quad [\neg\,(x) = ((\text{Chop}x) \oplus \underline{1})]]$

Special cases of the hypothesis and previous results:

0: $\neg\, \epsilon = x$ from H:x

1: $\neg\,(\text{Chop}x) \oplus \underline{0} = x$ from H:x

2: $\neg\,(\text{Chop}x) \oplus \underline{1} = x$ from H:x

3: $\epsilon = x \quad \vee \quad \neg\,\text{Parity}(\text{R}x) = 0 \quad \vee \quad (\text{Chop}x) \oplus \underline{0} = x$ from 251;x

4: $\epsilon = x \quad \vee \quad \neg\,\text{Parity}(\text{R}x) = 1 \quad \vee \quad (\text{Chop}x) \oplus \underline{1} = x$ from 252;x

5: $\text{Parity}(\text{R}x) = 0 \quad \vee \quad \text{Parity}(\text{R}x) = 1$ from 209;Rx

Inferences:

6: $\neg\,\text{Parity}(\text{R}x) = 0 \quad \vee \quad (\text{Chop}x) \oplus \underline{0} = x$ by
- 0: $\neg\, \epsilon = x$
- 3: $\epsilon = x \quad \vee \quad \neg\,\text{Parity}(\text{R}x) = 0 \quad \vee \quad (\text{Chop}x) \oplus \underline{0} = x$

7: $\neg\,\text{Parity}(\text{R}x) = 1 \quad \vee \quad (\text{Chop}x) \oplus \underline{1} = x$ by
- 0: $\neg\, \epsilon = x$
- 4: $\epsilon = x \quad \vee \quad \neg\,\text{Parity}(\text{R}x) = 1 \quad \vee \quad (\text{Chop}x) \oplus \underline{1} = x$

8: $\neg\,\text{Parity}(\text{R}x) = 0$ by
- 1: $\neg\,(\text{Chop}x) \oplus \underline{0} = x$
- 6: $\neg\,\text{Parity}(\text{R}x) = 0 \quad \vee \quad (\text{Chop}x) \oplus \underline{0} = x$

9: $\neg\,\text{Parity}(\text{R}x) = 1$ by
- 2: $\neg\,(\text{Chop}x) \oplus \underline{1} = x$
- 7: $\neg\,\text{Parity}(\text{R}x) = 1 \quad \vee \quad (\text{Chop}x) \oplus \underline{1} = x$

10: $\text{Parity}(\text{R}x) = 1$ by
- 8: $\neg\,\text{Parity}(\text{R}x) = 0$
- 5: $\text{Parity}(\text{R}x) = 0 \quad \vee \quad \text{Parity}(\text{R}x) = 1$

11: QEA by
- 9: $\neg\,\text{Parity}(\text{R}x) = 1$
- 10: $\text{Parity}(\text{R}x) = 1$