Proof of Theorem 252

The theorem to be proved is

$x \neq \epsilon \quad \& \quad \text{Parity R}x = 1 \quad \rightarrow \quad x = \text{Chop}\,x \oplus \underline{1}$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\,(x) = (\epsilon)] \quad \& \quad [(\text{Parity}(\text{R}x)) = (1)] \quad \& \quad [\neg\,(x) = ((\text{Chop}x) \oplus \underline{1})]]$

Special cases of the hypothesis and previous results:

0: $\neg\,\epsilon = x$ from H:x

1: $\text{Parity}(\text{R}x) = 1$ from H:x

2: $\neg\,(\text{Chop}x) \oplus \underline{1} = x$ from H:x

3: $\epsilon = x \quad \vee \quad \text{Q}((\text{Chop}x) \oplus \underline{1}) = \text{Q}x$ from 245;x

4: $\epsilon = x \quad \vee \quad \neg\,\text{Parity}(\text{R}x) = 1 \quad \vee \quad \text{R}((\text{Chop}x) \oplus \underline{1}) = \text{R}x$ from 250;x

5: $\neg\,\text{Q}((\text{Chop}x) \oplus \underline{1}) = \text{Q}x \quad \vee \quad \neg\,\text{R}((\text{Chop}x) \oplus \underline{1}) = \text{R}x \quad \vee \quad (\text{Chop}x) \oplus \underline{1} = x$ from 193;x;$(\text{Chop}x) \oplus \underline{1}$

Inferences:

6: $\text{Q}((\text{Chop}x) \oplus \underline{1}) = \text{Q}x$ by
0: $\neg\,\epsilon = x$
3: $\epsilon = x \quad \vee \quad \text{Q}((\text{Chop}x) \oplus \underline{1}) = \text{Q}x$

7: $\neg\,\text{Parity}(\text{R}x) = 1 \quad \vee \quad \text{R}((\text{Chop}x) \oplus \underline{1}) = \text{R}x$ by
0: $\neg\,\epsilon = x$
4: $\epsilon = x \quad \vee \quad \neg\,\text{Parity}(\text{R}x) = 1 \quad \vee \quad \text{R}((\text{Chop}x) \oplus \underline{1}) = \text{R}x$

8: $\text{R}((\text{Chop}x) \oplus \underline{1}) = \text{R}x$ by
1: $\text{Parity}(\text{R}x) = 1$
7: $\neg\,\text{Parity}(\text{R}x) = 1 \quad \vee \quad \text{R}((\text{Chop}x) \oplus \underline{1}) = \text{R}x$

9: $\neg\,\text{Q}((\text{Chop}x) \oplus \underline{1}) = \text{Q}x \quad \vee \quad \neg\,\text{R}((\text{Chop}x) \oplus \underline{1}) = \text{R}x$ by
2: $\neg\,(\text{Chop}x) \oplus \underline{1} = x$
5: $\neg\,\text{Q}((\text{Chop}x) \oplus \underline{1}) = \text{Q}x \quad \vee \quad \neg\,\text{R}((\text{Chop}x) \oplus \underline{1}) = \text{R}x \quad \vee \quad (\text{Chop}x) \oplus \underline{1} = x$

10: $\neg\,\text{R}((\text{Chop}x) \oplus \underline{1}) = \text{R}x$ by
6: $\text{Q}((\text{Chop}x) \oplus \underline{1}) = \text{Q}x$
9: $\neg\,\text{Q}((\text{Chop}x) \oplus \underline{1}) = \text{Q}x \quad \vee \quad \neg\,\text{R}((\text{Chop}x) \oplus \underline{1}) = \text{R}x$

11: QEA by

8: $\mathrm{R}((\mathrm{Chop}x) \oplus \underline{1}) = \mathrm{R}x$

10: $\neg\ \mathrm{R}((\mathrm{Chop}x) \oplus \underline{1}) = \mathrm{R}x$