Proof of Theorem 250

The theorem to be proved is

$x \neq \epsilon \quad \& \quad \text{Parity}\,\text{R}x = 1 \quad \rightarrow \quad \text{R}(\text{Chop}\,x \oplus \underline{1}) = \text{R}x$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\,(x) = (\epsilon)] \quad \& \quad [(\text{Parity}(\text{R}x)) = (1)] \quad \& \quad [\neg\,(\text{R}((\text{Chop}x) \oplus \underline{1})) = (\text{R}x)]]$

Special cases of the hypothesis and previous results:

0: $\neg\,\epsilon = x$ from H:x

1: $\text{Parity}(\text{R}x) = 1$ from H:x

2: $\neg\,\text{R}((\text{Chop}x) \oplus \underline{1}) = \text{R}x$ from H:x

3: $\epsilon = x \quad \vee \quad ((\text{Half}(\text{R}x)) \cdot 2) + 1 = \text{R}((\text{Chop}x) \oplus \underline{1})$ from 249;x

4: $\text{Half}(((\text{Half}(\text{R}x)) \cdot 2) + 1) = \text{Half}(\text{R}x)$ from 241;$\text{Half}(\text{R}x)$

5: $\text{Parity}(((\text{Half}(\text{R}x)) \cdot 2) + 1) = 1$ from 214;$\text{Half}(\text{R}x)$

6: $\neg\,\text{Parity}(\text{R}((\text{Chop}x) \oplus \underline{1})) = \text{Parity}(\text{R}x) \quad \vee \quad \neg\,\text{Half}(\text{R}((\text{Chop}x) \oplus \underline{1})) = \text{Half}(\text{R}x) \quad \vee \quad \text{R}((\text{Chop}x) \oplus \underline{1}) = \text{R}x$ from 246;$\text{R}((\text{Chop}x) \oplus \underline{1})$;$\text{R}x$

Equality substitutions:

7: $\neg\,\text{Parity}(\text{R}x) = 1 \quad \vee \quad \text{Parity}(\text{R}((\text{Chop}x) \oplus \underline{1})) = \text{Parity}(\text{R}x) \quad \vee \quad \neg\,\text{Parity}(\text{R}((\text{Chop}x) \oplus \underline{1})) = 1$

8: $\neg\,((\text{Half}(\text{R}x)) \cdot 2) + 1 = \text{R}((\text{Chop}x) \oplus \underline{1}) \quad \vee \quad \neg\,\text{Half}(((\text{Half}(\text{R}x)) \cdot 2) + 1) = \text{Half}(\text{R}x) \quad \vee \quad \text{Half}(\text{R}((\text{Chop}x) \oplus \underline{1})) = \text{Half}(\text{R}x)$

9: $\neg\,((\text{Half}(\text{R}x)) \cdot 2) + 1 = \text{R}((\text{Chop}x) \oplus \underline{1}) \quad \vee \quad \neg\,\text{Parity}(((\text{Half}(\text{R}x)) \cdot 2) + 1) = 1 \quad \vee \quad \text{Parity}(\text{R}((\text{Chop}x) \oplus \underline{1})) = 1$

Inferences:

10: $((\text{Half}(\text{R}x)) \cdot 2) + 1 = \text{R}((\text{Chop}x) \oplus \underline{1})$ by

0: $\neg\,\epsilon = x$

3: $\epsilon = x \quad \vee \quad ((\text{Half}(\text{R}x)) \cdot 2) + 1 = \text{R}((\text{Chop}x) \oplus \underline{1})$

11: $\text{Parity}(\text{R}((\text{Chop}x) \oplus \underline{1})) = \text{Parity}(\text{R}x) \quad \vee \quad \neg\,\text{Parity}(\text{R}((\text{Chop}x) \oplus \underline{1})) = 1$ by

1: $\text{Parity}(\text{R}x) = 1$

7: $\neg$ Parity(Rx) = 1 $\vee$ Parity(R((Chop$x$)$\oplus\underline{1}$)) = Parity(Rx) $\vee$ $\neg$ Parity(R((Chop$x$)$\oplus$ $\underline{1}$)) = 1

12: $\neg$ Parity(R((Chop$x$) $\oplus$ $\underline{1}$)) = Parity(Rx) $\vee$ $\neg$ Half(R((Chop$x$) $\oplus$ $\underline{1}$)) = Half(Rx) by

2: $\neg$ R((Chop$x$) $\oplus$ $\underline{1}$) = Rx

6: $\neg$ Parity(R((Chop$x$) $\oplus$ $\underline{1}$)) = Parity(Rx) $\vee$ $\neg$ Half(R((Chop$x$) $\oplus$ $\underline{1}$)) = Half(Rx) $\vee$ R((Chop$x$) $\oplus$ $\underline{1}$) = Rx

13: $\neg$ ((Half(Rx)) $\cdot$ 2) + 1 = R((Chop$x$) $\oplus$ $\underline{1}$) $\vee$ Half(R((Chop$x$) $\oplus$ $\underline{1}$)) = Half(Rx) by

4: Half(((Half(Rx)) $\cdot$ 2) + 1) = Half(Rx)

8: $\neg$ ((Half(Rx)) $\cdot$ 2) + 1 = R((Chop$x$) $\oplus$ $\underline{1}$) $\vee$ $\neg$ Half(((Half(Rx)) $\cdot$ 2) + 1) = Half(Rx) $\vee$ Half(R((Chop$x$) $\oplus$ $\underline{1}$)) = Half(Rx)

14: $\neg$ ((Half(Rx)) $\cdot$ 2) + 1 = R((Chop$x$) $\oplus$ $\underline{1}$) $\vee$ Parity(R((Chop$x$) $\oplus$ $\underline{1}$)) = 1 by

5: Parity(((Half(Rx)) $\cdot$ 2) + 1) = 1

9: $\neg$ ((Half(Rx)) $\cdot$ 2) + 1 = R((Chop$x$) $\oplus$ $\underline{1}$) $\vee$ $\neg$ Parity(((Half(Rx)) $\cdot$ 2) + 1) = 1 $\vee$ Parity(R((Chop$x$) $\oplus$ $\underline{1}$)) = 1

15: Half(R((Chop$x$) $\oplus$ $\underline{1}$)) = Half(Rx) by

10: ((Half(Rx)) $\cdot$ 2) + 1 = R((Chop$x$) $\oplus$ $\underline{1}$)

13: $\neg$ ((Half(Rx)) $\cdot$ 2) + 1 = R((Chop$x$) $\oplus$ $\underline{1}$) $\vee$ Half(R((Chop$x$) $\oplus$ $\underline{1}$)) = Half(Rx)

16: Parity(R((Chop$x$) $\oplus$ $\underline{1}$)) = 1 by

10: ((Half(Rx)) $\cdot$ 2) + 1 = R((Chop$x$) $\oplus$ $\underline{1}$)

14: $\neg$ ((Half(Rx)) $\cdot$ 2) + 1 = R((Chop$x$) $\oplus$ $\underline{1}$) $\vee$ Parity(R((Chop$x$) $\oplus$ $\underline{1}$)) = 1

17: $\neg$ Parity(R((Chop$x$) $\oplus$ $\underline{1}$)) = Parity(Rx) by

15: Half(R((Chop$x$) $\oplus$ $\underline{1}$)) = Half(Rx)

12: $\neg$ Parity(R((Chop$x$) $\oplus$ $\underline{1}$)) = Parity(Rx) $\vee$ $\neg$ Half(R((Chop$x$) $\oplus$ $\underline{1}$)) = Half(Rx)

18: Parity(R((Chop$x$) $\oplus$ $\underline{1}$)) = Parity(Rx) by

16: Parity(R((Chop$x$) $\oplus$ $\underline{1}$)) = 1

11: Parity(R((Chop$x$) $\oplus$ $\underline{1}$)) = Parity(Rx) $\vee$ $\neg$ Parity(R((Chop$x$) $\oplus$ $\underline{1}$)) = 1

19: QEA by

17: $\neg$ Parity(R((Chop$x$) $\oplus$ $\underline{1}$)) = Parity(Rx)

18: Parity(R((Chop$x$) $\oplus$ $\underline{1}$)) = Parity(Rx)