Proof of Theorem 249

The theorem to be proved is

$x \neq \epsilon \quad \rightarrow \quad \mathrm{R}(\mathrm{Chop}x \oplus \underline{1}) = \mathrm{Half}\ \mathrm{R}x \cdot 2 + 1$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\ (x) = (\epsilon)] \quad \& \quad [\neg\ (\mathrm{R}((\mathrm{Chop}x) \oplus \underline{1})) = (((\mathrm{Half}(\mathrm{R}x)) \cdot 2) + 1)]]$

Special cases of the hypothesis and previous results:

0: $\neg\ \epsilon = x$ from H:x

1: $\neg\ ((\mathrm{Half}(\mathrm{R}x)) \cdot 2) + 1 = \mathrm{R}((\mathrm{Chop}x) \oplus \underline{1})$ from H:x

2: $((\mathrm{R}(\mathrm{Chop}x)) \cdot (\mathrm{Q}\underline{1})) + (\mathrm{R}\underline{1}) = \mathrm{R}((\mathrm{Chop}x) \oplus \underline{1})$ from 180;Chop$x$;$\underline{1}$

3: $\mathrm{Q}\underline{1} = 2$ from 192

4: $\mathrm{R}\underline{1} = 1$ from 192

5: $\epsilon = x \quad \vee \quad \mathrm{R}(\mathrm{Chop}x) = \mathrm{Half}(\mathrm{R}x)$ from 239;x

Equality substitutions:

6: $\neg\ \mathrm{Q}\underline{1} = 2 \quad \vee \quad \neg\ ((\mathrm{R}(\mathrm{Chop}x)) \cdot (\mathrm{Q}\underline{1})) + (\mathrm{R}\underline{1}) = \mathrm{R}((\mathrm{Chop}x) \oplus \underline{1}) \quad \vee \quad ((\mathrm{R}(\mathrm{Chop}x)) \cdot (2)) + (\mathrm{R}\underline{1}) = \mathrm{R}((\mathrm{Chop}x) \oplus \underline{1})$

7: $\neg\ \mathrm{R}\underline{1} = 1 \quad \vee \quad \neg\ ((\mathrm{Half}(\mathrm{R}x)) \cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}((\mathrm{Chop}x) \oplus \underline{1}) \quad \vee \quad ((\mathrm{Half}(\mathrm{R}x)) \cdot 2) + (1) = \mathrm{R}((\mathrm{Chop}x) \oplus \underline{1})$

8: $\neg\ \mathrm{R}(\mathrm{Chop}x) = \mathrm{Half}(\mathrm{R}x) \quad \vee \quad \neg\ ((\mathrm{R}(\mathrm{Chop}x)) \cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}((\mathrm{Chop}x) \oplus \underline{1}) \quad \vee \quad ((\mathrm{Half}(\mathrm{R}x)) \cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}((\mathrm{Chop}x) \oplus \underline{1})$

Inferences:

9: $\mathrm{R}(\mathrm{Chop}x) = \mathrm{Half}(\mathrm{R}x)$ by

0: $\neg\ \epsilon = x$

5: $\epsilon = x \quad \vee \quad \mathrm{R}(\mathrm{Chop}x) = \mathrm{Half}(\mathrm{R}x)$

10: $\neg\ \mathrm{R}\underline{1} = 1 \quad \vee \quad \neg\ ((\mathrm{Half}(\mathrm{R}x)) \cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}((\mathrm{Chop}x) \oplus \underline{1})$ by

1: $\neg\ ((\mathrm{Half}(\mathrm{R}x)) \cdot 2) + 1 = \mathrm{R}((\mathrm{Chop}x) \oplus \underline{1})$

7: $\neg\ \mathrm{R}\underline{1} = 1 \quad \vee \quad \neg\ ((\mathrm{Half}(\mathrm{R}x)) \cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}((\mathrm{Chop}x) \oplus \underline{1}) \quad \vee \quad ((\mathrm{Half}(\mathrm{R}x)) \cdot 2) + 1 = \mathrm{R}((\mathrm{Chop}x) \oplus \underline{1})$

11: $\neg\ \mathrm{Q}\underline{1} = 2 \quad \vee \quad ((\mathrm{R}(\mathrm{Chop}x)) \cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}((\mathrm{Chop}x) \oplus \underline{1})$ by

2: $((\mathrm{R}(\mathrm{Chop}x)) \cdot (\mathrm{Q}\underline{1})) + (\mathrm{R}\underline{1}) = \mathrm{R}((\mathrm{Chop}x) \oplus \underline{1})$

6: $\neg\ \mathrm{Q}\underline{1} = 2 \quad \vee \quad \neg\ ((\mathrm{R}(\mathrm{Chop}x)) \cdot (\mathrm{Q}\underline{1})) + (\mathrm{R}\underline{1}) = \mathrm{R}((\mathrm{Chop}x) \oplus \underline{1}) \quad \vee \quad ((\mathrm{R}(\mathrm{Chop}x)) \cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}((\mathrm{Chop}x) \oplus \underline{1})$

12: $((\mathrm{R}(\mathrm{Chop}x)) \cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}((\mathrm{Chop}x) \oplus \underline{1})$ by

3: $\mathrm{Q}\underline{1} = 2$

11: $\neg\ \mathrm{Q}\underline{1} = 2 \quad \vee \quad ((\mathrm{R}(\mathrm{Chop}x)) \cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}((\mathrm{Chop}x) \oplus \underline{1})$

13: $\neg\ ((\mathrm{Half}(\mathrm{R}x)) \cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}((\mathrm{Chop}x) \oplus \underline{1})$ by

4: $\mathrm{R}\underline{1} = 1$

10: $\neg\ \mathrm{R}\underline{1} = 1 \quad \vee \quad \neg\ ((\mathrm{Half}(\mathrm{R}x)) \cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}((\mathrm{Chop}x) \oplus \underline{1})$

14: $\neg\ ((\mathrm{R}(\mathrm{Chop}x)) \cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}((\mathrm{Chop}x) \oplus \underline{1}) \quad \vee \quad ((\mathrm{Half}(\mathrm{R}x)) \cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}((\mathrm{Chop}x) \oplus \underline{1})$ by

9: $\mathrm{R}(\mathrm{Chop}x) = \mathrm{Half}(\mathrm{R}x)$

8: $\neg\ \mathrm{R}(\mathrm{Chop}x) = \mathrm{Half}(\mathrm{R}x) \quad \vee \quad \neg\ ((\mathrm{R}(\mathrm{Chop}x)) \cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}((\mathrm{Chop}x) \oplus \underline{1}) \quad \vee \quad ((\mathrm{Half}(\mathrm{R}x)) \cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}((\mathrm{Chop}x) \oplus \underline{1})$

15: $((\mathrm{Half}(\mathrm{R}x)) \cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}((\mathrm{Chop}x) \oplus \underline{1})$ by

12: $((\mathrm{R}(\mathrm{Chop}x)) \cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}((\mathrm{Chop}x) \oplus \underline{1})$

14: $\neg\ ((\mathrm{R}(\mathrm{Chop}x)) \cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}((\mathrm{Chop}x) \oplus \underline{1}) \quad \vee \quad ((\mathrm{Half}(\mathrm{R}x)) \cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}((\mathrm{Chop}x) \oplus \underline{1})$

16: QEA by

13: $\neg\ ((\mathrm{Half}(\mathrm{R}x)) \cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}((\mathrm{Chop}x) \oplus \underline{1})$

15: $((\mathrm{Half}(\mathrm{R}x)) \cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}((\mathrm{Chop}x) \oplus \underline{1})$