Proof of Theorem 246

The theorem to be proved is

Parity x = Parity y & Half x = Half y → $x = y$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) [[(Parity$x$) = (Parity$y$)] & [(Half$x$) = (Half$y$)] & [¬ (x) = (y)]]

Special cases of the hypothesis and previous results:

0: Parity$y$ = Parity$x$ from H:x:y

1: Half$y$ = Half$x$ from H:x:y

2: ¬ $y = x$ from H:x:y

3: ¬ Parity$x$ = 0 ∨ 2 · (Half$x$) = x from 224;x

4: ¬ Parity$y$ = 0 ∨ 2 · (Half$y$) = y from 224;y

5: ¬ Parity$x$ = 1 ∨ (2 · (Half$x$)) + 1 = x from 225;x

6: ¬ Parity$y$ = 1 ∨ (2 · (Half$y$)) + 1 = y from 225;y

7: Parity$x$ = 0 ∨ Parity$x$ = 1 from 209;x

Equality substitutions:

8: ¬ Parity$y$ = Parity$x$ ∨ Parity$y$ = 0 ∨ ¬ Parity$x$ = 0

9: ¬ Parity$y$ = Parity$x$ ∨ Parity$y$ = 1 ∨ ¬ Parity$x$ = 1

10: ¬ Half$y$ = Half$x$ ∨ ¬ 2 · (Half$y$) = y ∨ 2 · (Half$x$) = y

11: ¬ Half$y$ = Half$x$ ∨ ¬ (2 · (Half$y$)) + 1 = y ∨ (2 · (Half$x$)) + 1 = y

12: ¬ 2 · (Half$x$) = x ∨ ¬ 2 · (Half$x$) = y ∨ $x = y$

13: ¬ (2 · (Half$x$)) + 1 = x ∨ ¬ (2 · (Half$x$)) + 1 = y ∨ $x = y$

Inferences:

14: Parity$y$ = 0 ∨ ¬ Parity$x$ = 0 by

0: Parity$y$ = Parity$x$

8: ¬ Parity$y$ = Parity$x$ ∨ Parity$y$ = 0 ∨ ¬ Parity$x$ = 0

15: $\text{Parity}y = 1 \quad \vee \quad \neg\ \text{Parity}x = 1$ by

0: $\text{Parity}y = \text{Parity}x$

9: $\neg\ \text{Parity}y = \text{Parity}x \quad \vee \quad \text{Parity}y = 1 \quad \vee \quad \neg\ \text{Parity}x = 1$

16: $\neg\ 2 \cdot (\text{Half}y) = y \quad \vee \quad 2 \cdot (\text{Half}x) = y$ by

1: $\text{Half}y = \text{Half}x$

10: $\neg\ \text{Half}y = \text{Half}x \quad \vee \quad \neg\ 2 \cdot (\text{Half}y) = y \quad \vee \quad 2 \cdot (\text{Half}x) = y$

17: $\neg\ (2 \cdot (\text{Half}y)) + 1 = y \quad \vee \quad (2 \cdot (\text{Half}x)) + 1 = y$ by

1: $\text{Half}y = \text{Half}x$

11: $\neg\ \text{Half}y = \text{Half}x \quad \vee \quad \neg\ (2 \cdot (\text{Half}y)) + 1 = y \quad \vee \quad (2 \cdot (\text{Half}x)) + 1 = y$

18: $\neg\ 2 \cdot (\text{Half}x) = x \quad \vee \quad \neg\ 2 \cdot (\text{Half}x) = y$ by

2: $\neg\ y = x$

12: $\neg\ 2 \cdot (\text{Half}x) = x \quad \vee \quad \neg\ 2 \cdot (\text{Half}x) = y \quad \vee \quad y = x$

19: $\neg\ (2 \cdot (\text{Half}x)) + 1 = x \quad \vee \quad \neg\ (2 \cdot (\text{Half}x)) + 1 = y$ by

2: $\neg\ y = x$

13: $\neg\ (2 \cdot (\text{Half}x)) + 1 = x \quad \vee \quad \neg\ (2 \cdot (\text{Half}x)) + 1 = y \quad \vee \quad y = x$

CLAIM: $\text{Parity}x = 0$ Suppose not. Then

20: $\neg\ \text{Parity}x = 0$

21: $\text{Parity}x = 1$ by

20: $\neg\ \text{Parity}x = 0$

7: $\text{Parity}x = 0 \quad \vee \quad \text{Parity}x = 1$

22: $(2 \cdot (\text{Half}x)) + 1 = x$ by

21: $\text{Parity}x = 1$

5: $\neg\ \text{Parity}x = 1 \quad \vee \quad (2 \cdot (\text{Half}x)) + 1 = x$

23: $\text{Parity}y = 1$ by

21: $\text{Parity}x = 1$

15: $\text{Parity}y = 1 \quad \vee \quad \neg\ \text{Parity}x = 1$

24: $\neg\ (2 \cdot (\text{Half}x)) + 1 = y$ by

22: $(2 \cdot (\text{Half}x)) + 1 = x$

19: $\neg\ (2 \cdot (\text{Half}x)) + 1 = x \quad \vee \quad \neg\ (2 \cdot (\text{Half}x)) + 1 = y$

25: $(2 \cdot (\text{Half}y)) + 1 = y$ by

23: $\text{Parity}y = 1$

6: $\neg\ \text{Parity}y = 1 \quad \vee \quad (2 \cdot (\text{Half}y)) + 1 = y$

26: $\neg\,(2\cdot(\mathrm{Half}y))+1=y$ by

24: $\neg\,(2\cdot(\mathrm{Half}x))+1=y$

17: $\neg\,(2\cdot(\mathrm{Half}y))+1=y \quad\vee\quad (2\cdot(\mathrm{Half}x))+1=y$

27: $\mathrm{Parity}x=0$ The CLAIM is proved, and 20–26 will not be used after this:

25: $(2\cdot(\mathrm{Half}y))+1=y$

26: $\neg\,(2\cdot(\mathrm{Half}y))+1=y$

28: $2\cdot(\mathrm{Half}x)=x$ by

27: $\mathrm{Parity}x=0$

3: $\neg\,\mathrm{Parity}x=0 \quad\vee\quad 2\cdot(\mathrm{Half}x)=x$

29: $\mathrm{Parity}y=0$ by

27: $\mathrm{Parity}x=0$

14: $\mathrm{Parity}y=0 \quad\vee\quad \neg\,\mathrm{Parity}x=0$

30: $\neg\,2\cdot(\mathrm{Half}x)=y$ by

28: $2\cdot(\mathrm{Half}x)=x$

18: $\neg\,2\cdot(\mathrm{Half}x)=x \quad\vee\quad \neg\,2\cdot(\mathrm{Half}x)=y$

31: $2\cdot(\mathrm{Half}y)=y$ by

29: $\mathrm{Parity}y=0$

4: $\neg\,\mathrm{Parity}y=0 \quad\vee\quad 2\cdot(\mathrm{Half}y)=y$

32: $\neg\,2\cdot(\mathrm{Half}y)=y$ by

30: $\neg\,2\cdot(\mathrm{Half}x)=y$

16: $\neg\,2\cdot(\mathrm{Half}y)=y \quad\vee\quad 2\cdot(\mathrm{Half}x)=y$

33: QEA by

31: $2\cdot(\mathrm{Half}y)=y$

32: $\neg\,2\cdot(\mathrm{Half}y)=y$