Proof of Theorem 245

The theorem to be proved is

$$x \neq \epsilon \quad \rightarrow \quad Q(\text{Chop } x \oplus \underline{0}) = Qx \quad \& \quad Q(\text{Chop } x \oplus \underline{1}) = Qx$$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $\quad [[\neg (x) = (\epsilon)] \quad \& \quad [\neg (Q((\text{Chop}x) \oplus \underline{0})) = (Qx) \quad \vee \quad \neg (Q((\text{Chop}x) \oplus \underline{1})) = (Qx)]]$

Special cases of the hypothesis and previous results:

0: $\quad \neg \, \epsilon = x \qquad$ from $\quad$ H:x

1: $\quad \neg \, Q((\text{Chop}x) \oplus \underline{0}) = Qx \quad \vee \quad \neg \, Q((\text{Chop}x) \oplus \underline{1}) = Qx \qquad$ from $\quad$ H:x

2: $\quad (Q(\text{Chop}x)) \cdot (Q\underline{0}) = Q((\text{Chop}x) \oplus \underline{0}) \qquad$ from $\quad$ 180;Chop$x$;$\underline{0}$

3: $\quad (Q(\text{Chop}x)) \cdot (Q\underline{1}) = Q((\text{Chop}x) \oplus \underline{1}) \qquad$ from $\quad$ 180;Chop$x$;$\underline{1}$

4: $\quad Q\underline{0} = 2 \qquad$ from $\quad$ 191

5: $\quad Q\underline{1} = 2 \qquad$ from $\quad$ 192

6: $\quad Qx$ is a power of two $\qquad$ from $\quad$ 158;x

7: $\quad \neg \, Qx = 1 \quad \vee \quad \epsilon = x \qquad$ from $\quad$ 203;x

8: $\quad \neg \, Qx$ is a power of two $\quad \vee \quad Qx = 1 \quad \vee \quad \text{Parity}(Qx) = 0 \qquad$ from $\quad$ 236;Qx

9: $\quad \neg \, \text{Parity}(Qx) = 0 \quad \vee \quad 2 \cdot (\text{Half}(Qx)) = Qx \qquad$ from $\quad$ 224;Qx

10: $\quad \epsilon = x \quad \vee \quad Q(\text{Chop}x) = \text{Half}(Qx) \qquad$ from $\quad$ 239;x

11: $\quad (Q(\text{Chop}x)) \cdot 2 = 2 \cdot (Q(\text{Chop}x)) \qquad$ from $\quad$ 105;2;$Q(\text{Chop}x)$

Equality substitutions:

12: $\quad \neg \, Q\underline{0} = 2 \quad \vee \quad \neg \, (Q(\text{Chop}x)) \cdot (Q\underline{0}) = Q((\text{Chop}x) \oplus \underline{0}) \quad \vee \quad (Q(\text{Chop}x)) \cdot (2) = Q((\text{Chop}x) \oplus \underline{0})$

13: $\quad \neg \, Q\underline{1} = 2 \quad \vee \quad \neg \, (Q(\text{Chop}x)) \cdot (Q\underline{1}) = Q((\text{Chop}x) \oplus \underline{1}) \quad \vee \quad (Q(\text{Chop}x)) \cdot (2) = Q((\text{Chop}x) \oplus \underline{1})$

14: $\quad \neg \, Q((\text{Chop}x) \oplus \underline{0}) = (Q(\text{Chop}x)) \cdot 2 \quad \vee \quad Q((\text{Chop}x) \oplus \underline{0}) = 2 \cdot (\text{Half}(Qx)) \quad \vee \quad \neg \, (Q(\text{Chop}x)) \cdot 2 = 2 \cdot (\text{Half}(Qx))$

15: $\quad \neg \, Q((\text{Chop}x) \oplus \underline{1}) = (Q(\text{Chop}x)) \cdot 2 \quad \vee \quad Q((\text{Chop}x) \oplus \underline{1}) = 2 \cdot (\text{Half}(Qx)) \quad \vee \quad \neg \, (Q(\text{Chop}x)) \cdot 2 = 2 \cdot (\text{Half}(Qx))$

16: $\neg\ Qx = 2 \cdot (\text{Half}(Qx))\ \lor\ Q((\text{Chop}x) \oplus \underline{0}) = (Qx)\ \lor\ \neg\ Q((\text{Chop}x) \oplus \underline{0}) = (2 \cdot (\text{Half}(Qx)))$

17: $\neg\ Qx = 2 \cdot (\text{Half}(Qx))\ \lor\ Q((\text{Chop}x) \oplus \underline{1}) = (Qx)\ \lor\ \neg\ Q((\text{Chop}x) \oplus \underline{1}) = (2 \cdot (\text{Half}(Qx)))$

18: $\neg\ \text{Half}(Qx) = Q(\text{Chop}x)\ \lor\ (Q(\text{Chop}x)) \cdot 2 = 2 \cdot ((\text{Half}(Qx)))\ \lor\ \neg\ (Q(\text{Chop}x)) \cdot 2 = 2 \cdot ((Q(\text{Chop}x)))$

Inferences:

19: $\neg\ Qx = 1$ by

    0: $\neg\ \epsilon = x$

    7: $\neg\ Qx = 1\ \lor\ \epsilon = x$

20: $Q(\text{Chop}x) = \text{Half}(Qx)$ by

    0: $\neg\ \epsilon = x$

    10: $\epsilon = x\ \lor\ Q(\text{Chop}x) = \text{Half}(Qx)$

21: $\neg\ Q\underline{0} = 2\ \lor\ Q((\text{Chop}x) \oplus \underline{0}) = (Q(\text{Chop}x)) \cdot 2$ by

    2: $(Q(\text{Chop}x)) \cdot (Q\underline{0}) = Q((\text{Chop}x) \oplus \underline{0})$

    12: $\neg\ Q\underline{0} = 2\ \lor\ \neg\ (Q(\text{Chop}x)) \cdot (Q\underline{0}) = Q((\text{Chop}x) \oplus \underline{0})\ \lor\ Q((\text{Chop}x) \oplus \underline{0}) = (Q(\text{Chop}x)) \cdot 2$

22: $\neg\ Q\underline{1} = 2\ \lor\ Q((\text{Chop}x) \oplus \underline{1}) = (Q(\text{Chop}x)) \cdot 2$ by

    3: $(Q(\text{Chop}x)) \cdot (Q\underline{1}) = Q((\text{Chop}x) \oplus \underline{1})$

    13: $\neg\ Q\underline{1} = 2\ \lor\ \neg\ (Q(\text{Chop}x)) \cdot (Q\underline{1}) = Q((\text{Chop}x) \oplus \underline{1})\ \lor\ Q((\text{Chop}x) \oplus \underline{1}) = (Q(\text{Chop}x)) \cdot 2$

23: $Q((\text{Chop}x) \oplus \underline{0}) = (Q(\text{Chop}x)) \cdot 2$ by

    4: $Q\underline{0} = 2$

    21: $\neg\ Q\underline{0} = 2\ \lor\ Q((\text{Chop}x) \oplus \underline{0}) = (Q(\text{Chop}x)) \cdot 2$

24: $Q((\text{Chop}x) \oplus \underline{1}) = (Q(\text{Chop}x)) \cdot 2$ by

    5: $Q\underline{1} = 2$

    22: $\neg\ Q\underline{1} = 2\ \lor\ Q((\text{Chop}x) \oplus \underline{1}) = (Q(\text{Chop}x)) \cdot 2$

25: $Qx = 1\ \lor\ \text{Parity}(Qx) = 0$ by

    6: Qx is a power of two

    8: $\neg\ Qx$ is a power of two $\ \lor\ Qx = 1\ \lor\ \text{Parity}(Qx) = 0$

26: $\neg\, Q(\text{Chop}x) = \text{Half}(Qx) \quad \vee \quad (Q(\text{Chop}x)) \cdot 2 = 2 \cdot (\text{Half}(Qx))$ by

    11: $(Q(\text{Chop}x)) \cdot 2 = 2 \cdot (Q(\text{Chop}x))$

    18: $\neg\, Q(\text{Chop}x) = \text{Half}(Qx) \quad \vee \quad (Q(\text{Chop}x)) \cdot 2 = 2 \cdot (\text{Half}(Qx)) \quad \vee \quad \neg\, (Q(\text{Chop}x)) \cdot 2 = 2 \cdot (Q(\text{Chop}x))$

27: $\text{Parity}(Qx) = 0$ by

    19: $\neg\, Qx = 1$

    25: $Qx = 1 \quad \vee \quad \text{Parity}(Qx) = 0$

28: $(Q(\text{Chop}x)) \cdot 2 = 2 \cdot (\text{Half}(Qx))$ by

    20: $Q(\text{Chop}x) = \text{Half}(Qx)$

    26: $\neg\, Q(\text{Chop}x) = \text{Half}(Qx) \quad \vee \quad (Q(\text{Chop}x)) \cdot 2 = 2 \cdot (\text{Half}(Qx))$

29: $Q((\text{Chop}x) \oplus \underline{0}) = 2 \cdot (\text{Half}(Qx)) \quad \vee \quad \neg\, (Q(\text{Chop}x)) \cdot 2 = 2 \cdot (\text{Half}(Qx))$ by

    23: $Q((\text{Chop}x) \oplus \underline{0}) = (Q(\text{Chop}x)) \cdot 2$

    14: $\neg\, Q((\text{Chop}x) \oplus \underline{0}) = (Q(\text{Chop}x)) \cdot 2 \quad \vee \quad Q((\text{Chop}x) \oplus \underline{0}) = 2 \cdot (\text{Half}(Qx))$ $\vee \quad \neg\, (Q(\text{Chop}x)) \cdot 2 = 2 \cdot (\text{Half}(Qx))$

30: $Q((\text{Chop}x) \oplus \underline{1}) = 2 \cdot (\text{Half}(Qx)) \quad \vee \quad \neg\, (Q(\text{Chop}x)) \cdot 2 = 2 \cdot (\text{Half}(Qx))$ by

    24: $Q((\text{Chop}x) \oplus \underline{1}) = (Q(\text{Chop}x)) \cdot 2$

    15: $\neg\, Q((\text{Chop}x) \oplus \underline{1}) = (Q(\text{Chop}x)) \cdot 2 \quad \vee \quad Q((\text{Chop}x) \oplus \underline{1}) = 2 \cdot (\text{Half}(Qx))$ $\vee \quad \neg\, (Q(\text{Chop}x)) \cdot 2 = 2 \cdot (\text{Half}(Qx))$

31: $2 \cdot (\text{Half}(Qx)) = Qx$ by

    27: $\text{Parity}(Qx) = 0$

    9: $\neg\, \text{Parity}(Qx) = 0 \quad \vee \quad 2 \cdot (\text{Half}(Qx)) = Qx$

32: $Q((\text{Chop}x) \oplus \underline{0}) = 2 \cdot (\text{Half}(Qx))$ by

    28: $(Q(\text{Chop}x)) \cdot 2 = 2 \cdot (\text{Half}(Qx))$

    29: $Q((\text{Chop}x) \oplus \underline{0}) = 2 \cdot (\text{Half}(Qx)) \quad \vee \quad \neg\, (Q(\text{Chop}x)) \cdot 2 = 2 \cdot (\text{Half}(Qx))$

33: $Q((\text{Chop}x) \oplus \underline{1}) = 2 \cdot (\text{Half}(Qx))$ by

    28: $(Q(\text{Chop}x)) \cdot 2 = 2 \cdot (\text{Half}(Qx))$

    30: $Q((\text{Chop}x) \oplus \underline{1}) = 2 \cdot (\text{Half}(Qx)) \quad \vee \quad \neg\, (Q(\text{Chop}x)) \cdot 2 = 2 \cdot (\text{Half}(Qx))$

34: $Q((\text{Chop}x) \oplus \underline{0}) = Qx \quad \vee \quad \neg\, Q((\text{Chop}x) \oplus \underline{0}) = 2 \cdot (\text{Half}(Qx))$ by

    31: $2 \cdot (\text{Half}(Qx)) = Qx$

    16: $\neg\, 2 \cdot (\text{Half}(Qx)) = Qx \quad \vee \quad Q((\text{Chop}x) \oplus \underline{0}) = Qx \quad \vee \quad \neg\, Q((\text{Chop}x) \oplus \underline{0}) = 2 \cdot (\text{Half}(Qx))$

35: $Q((\text{Chop}x) \oplus \underline{1}) = Qx \quad \vee \quad \neg\, Q((\text{Chop}x) \oplus \underline{1}) = 2 \cdot (\text{Half}(Qx))$ by

    31: $2 \cdot (\text{Half}(Qx)) = Qx$

17: $\neg\ 2 \cdot (\mathrm{Half}(\mathrm{Q}x)) = \mathrm{Q}x \quad \vee \quad \mathrm{Q}((\mathrm{Chop}x) \oplus \underline{1}) = \mathrm{Q}x \quad \vee \quad \neg\ \mathrm{Q}((\mathrm{Chop}x) \oplus \underline{1}) = 2 \cdot (\mathrm{Half}(\mathrm{Q}x))$

36: $\mathrm{Q}((\mathrm{Chop}x) \oplus \underline{0}) = \mathrm{Q}x \qquad$ by

    32: $\mathrm{Q}((\mathrm{Chop}x) \oplus \underline{0}) = 2 \cdot (\mathrm{Half}(\mathrm{Q}x))$

    34: $\mathrm{Q}((\mathrm{Chop}x) \oplus \underline{0}) = \mathrm{Q}x \quad \vee \quad \neg\ \mathrm{Q}((\mathrm{Chop}x) \oplus \underline{0}) = 2 \cdot (\mathrm{Half}(\mathrm{Q}x))$

37: $\mathrm{Q}((\mathrm{Chop}x) \oplus \underline{1}) = \mathrm{Q}x \qquad$ by

    33: $\mathrm{Q}((\mathrm{Chop}x) \oplus \underline{1}) = 2 \cdot (\mathrm{Half}(\mathrm{Q}x))$

    35: $\mathrm{Q}((\mathrm{Chop}x) \oplus \underline{1}) = \mathrm{Q}x \quad \vee \quad \neg\ \mathrm{Q}((\mathrm{Chop}x) \oplus \underline{1}) = 2 \cdot (\mathrm{Half}(\mathrm{Q}x))$

38: $\neg\ \mathrm{Q}((\mathrm{Chop}x) \oplus \underline{1}) = \mathrm{Q}x \qquad$ by

    36: $\mathrm{Q}((\mathrm{Chop}x) \oplus \underline{0}) = \mathrm{Q}x$

    1: $\neg\ \mathrm{Q}((\mathrm{Chop}x) \oplus \underline{0}) = \mathrm{Q}x \quad \vee \quad \neg\ \mathrm{Q}((\mathrm{Chop}x) \oplus \underline{1}) = \mathrm{Q}x$

39: $QEA \qquad$ by

    37: $\mathrm{Q}((\mathrm{Chop}x) \oplus \underline{1}) = \mathrm{Q}x$

    38: $\neg\ \mathrm{Q}((\mathrm{Chop}x) \oplus \underline{1}) = \mathrm{Q}x$