Proof of Theorem 244

The theorem to be proved is

$\mathrm{Chop}(x \oplus \underline{1}) = x$ ⋆

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\ (\mathrm{Chop}(x \oplus \underline{1})) = (x)]]$

Special cases of the hypothesis and previous results:

0: $\neg\ \mathrm{Chop}(x \oplus \underline{1}) = x$ from H:x

1: $\neg\ x \oplus \underline{1} = \epsilon$ from 242;x

2: $x \oplus \underline{1} = \epsilon \quad \vee \quad \mathrm{Q}(\mathrm{Chop}(x \oplus \underline{1})) = \mathrm{Half}(\mathrm{Q}(x \oplus \underline{1}))$ from 239;$x \oplus \underline{1}$

3: $x \oplus \underline{1} = \epsilon \quad \vee \quad \mathrm{R}(\mathrm{Chop}(x \oplus \underline{1})) = \mathrm{Half}(\mathrm{R}(x \oplus \underline{1}))$ from 239;$x \oplus \underline{1}$

4: $(\mathrm{Q}x) \cdot (\mathrm{Q}\underline{1}) = \mathrm{Q}(x \oplus \underline{1})$ from 180;x;$\underline{1}$

5: $((\mathrm{R}x) \cdot (\mathrm{Q}\underline{1})) + (\mathrm{R}\underline{1}) = \mathrm{R}(x \oplus \underline{1})$ from 180;x;$\underline{1}$

6: $\mathrm{Q}\underline{1} = 2$ from 192

7: $\mathrm{R}\underline{1} = 1$ from 192

8: $\mathrm{Half}((\mathrm{Q}x) \cdot 2) = \mathrm{Q}x$ from 241;$\mathrm{Q}x$

9: $\mathrm{Half}(((\mathrm{R}x) \cdot 2) + 1) = \mathrm{R}x$ from 241;$\mathrm{R}x$

10: $\neg\ \mathrm{Q}(\mathrm{Chop}(x \oplus \underline{1})) = \mathrm{Q}x \quad \vee \quad \neg\ \mathrm{R}(\mathrm{Chop}(x \oplus \underline{1})) = \mathrm{R}x \quad \vee \quad \mathrm{Chop}(x \oplus \underline{1}) = x$ from 193;x;$\mathrm{Chop}(x \oplus \underline{1})$

Equality substitutions:

11: $\neg\ \mathrm{Q}(\mathrm{Chop}(x \oplus \underline{1})) = \mathrm{Half}(\mathrm{Q}(x \oplus \underline{1})) \quad \vee \quad \mathrm{Q}(\mathrm{Chop}(x \oplus \underline{1})) = \mathrm{Q}x \quad \vee \quad \neg\ \mathrm{Half}(\mathrm{Q}(x \oplus \underline{1})) = \mathrm{Q}x$

12: $\neg\ \mathrm{R}(\mathrm{Chop}(x \oplus \underline{1})) = \mathrm{Half}(\mathrm{R}(x \oplus \underline{1})) \quad \vee \quad \mathrm{R}(\mathrm{Chop}(x \oplus \underline{1})) = \mathrm{R}x \quad \vee \quad \neg\ \mathrm{Half}(\mathrm{R}(x \oplus \underline{1})) = \mathrm{R}x$

13: $\neg\ \mathrm{Q}\underline{1} = 2 \quad \vee \quad \neg\ (\mathrm{Q}x) \cdot (\mathrm{Q}\underline{1}) = \mathrm{Q}(x \oplus \underline{1}) \quad \vee \quad (\mathrm{Q}x) \cdot (2) = \mathrm{Q}(x \oplus \underline{1})$

14: $\neg\ \mathrm{Q}\underline{1} = 2 \quad \vee \quad \neg\ ((\mathrm{R}x) \cdot (\mathrm{Q}\underline{1})) + (\mathrm{R}\underline{1}) = \mathrm{R}(x \oplus \underline{1}) \quad \vee \quad ((\mathrm{R}x) \cdot (2)) + (\mathrm{R}\underline{1}) = \mathrm{R}(x \oplus \underline{1})$

15: $\neg\ \mathrm{R}\underline{1} = 1 \quad \vee \quad \neg\ ((\mathrm{R}x) \cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}(x \oplus \underline{1}) \quad \vee \quad ((\mathrm{R}x) \cdot 2) + (1) = \mathrm{R}(x \oplus \underline{1})$

16: $\neg\ \mathrm{Q}(x \oplus \underline{1}) = (\mathrm{Q}x) \cdot 2 \quad \vee \quad \mathrm{Half}(\mathrm{Q}(x \oplus \underline{1})) = \mathrm{Q}x \quad \vee \quad \neg\ \mathrm{Half}((\mathrm{Q}x) \cdot 2) = \mathrm{Q}x$

17: $\neg\, ((\mathrm{R}x)\cdot 2)+1 = \mathrm{R}(x\oplus \underline{1}) \quad \vee \quad \neg\, \mathrm{Half}(((\mathrm{R}x)\cdot 2)+1) = \mathrm{R}x \quad \vee \quad \mathrm{Half}(\mathrm{R}(x\oplus \underline{1})) = \mathrm{R}x$

Inferences:

18: $\neg\, \mathrm{Q}(\mathrm{Chop}(x\oplus \underline{1})) = \mathrm{Q}x \quad \vee \quad \neg\, \mathrm{R}(\mathrm{Chop}(x\oplus \underline{1})) = \mathrm{R}x$ by
0: $\neg\, \mathrm{Chop}(x\oplus \underline{1}) = x$
10: $\neg\, \mathrm{Q}(\mathrm{Chop}(x\oplus \underline{1})) = \mathrm{Q}x \quad \vee \quad \neg\, \mathrm{R}(\mathrm{Chop}(x\oplus \underline{1})) = \mathrm{R}x \quad \vee \quad \mathrm{Chop}(x\oplus \underline{1}) = x$

19: $\mathrm{Q}(\mathrm{Chop}(x\oplus \underline{1})) = \mathrm{Half}(\mathrm{Q}(x\oplus \underline{1}))$ by
1: $\neg\, x\oplus \underline{1} = \epsilon$
2: $x\oplus \underline{1} = \epsilon \quad \vee \quad \mathrm{Q}(\mathrm{Chop}(x\oplus \underline{1})) = \mathrm{Half}(\mathrm{Q}(x\oplus \underline{1}))$

20: $\mathrm{R}(\mathrm{Chop}(x\oplus \underline{1})) = \mathrm{Half}(\mathrm{R}(x\oplus \underline{1}))$ by
1: $\neg\, x\oplus \underline{1} = \epsilon$
3: $x\oplus \underline{1} = \epsilon \quad \vee \quad \mathrm{R}(\mathrm{Chop}(x\oplus \underline{1})) = \mathrm{Half}(\mathrm{R}(x\oplus \underline{1}))$

21: $\neg\, \mathrm{Q}\underline{1} = 2 \quad \vee \quad \mathrm{Q}(x\oplus \underline{1}) = (\mathrm{Q}x)\cdot 2$ by
4: $(\mathrm{Q}x)\cdot(\mathrm{Q}\underline{1}) = \mathrm{Q}(x\oplus \underline{1})$
13: $\neg\, \mathrm{Q}\underline{1} = 2 \quad \vee \quad \neg\, (\mathrm{Q}x)\cdot(\mathrm{Q}\underline{1}) = \mathrm{Q}(x\oplus \underline{1}) \quad \vee \quad \mathrm{Q}(x\oplus \underline{1}) = (\mathrm{Q}x)\cdot 2$

22: $\neg\, \mathrm{Q}\underline{1} = 2 \quad \vee \quad ((\mathrm{R}x)\cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}(x\oplus \underline{1})$ by
5: $((\mathrm{R}x)\cdot(\mathrm{Q}\underline{1})) + (\mathrm{R}\underline{1}) = \mathrm{R}(x\oplus \underline{1})$
14: $\neg\, \mathrm{Q}\underline{1} = 2 \quad \vee \quad \neg\, ((\mathrm{R}x)\cdot(\mathrm{Q}\underline{1})) + (\mathrm{R}\underline{1}) = \mathrm{R}(x\oplus \underline{1}) \quad \vee \quad ((\mathrm{R}x)\cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}(x\oplus \underline{1})$

23: $\mathrm{Q}(x\oplus \underline{1}) = (\mathrm{Q}x)\cdot 2$ by
6: $\mathrm{Q}\underline{1} = 2$
21: $\neg\, \mathrm{Q}\underline{1} = 2 \quad \vee \quad \mathrm{Q}(x\oplus \underline{1}) = (\mathrm{Q}x)\cdot 2$

24: $((\mathrm{R}x)\cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}(x\oplus \underline{1})$ by
6: $\mathrm{Q}\underline{1} = 2$
22: $\neg\, \mathrm{Q}\underline{1} = 2 \quad \vee \quad ((\mathrm{R}x)\cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}(x\oplus \underline{1})$

25: $\neg\, ((\mathrm{R}x)\cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}(x\oplus \underline{1}) \quad \vee \quad ((\mathrm{R}x)\cdot 2) + 1 = \mathrm{R}(x\oplus \underline{1})$ by
7: $\mathrm{R}\underline{1} = 1$
15: $\neg\, \mathrm{R}\underline{1} = 1 \quad \vee \quad \neg\, ((\mathrm{R}x)\cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}(x\oplus \underline{1}) \quad \vee \quad ((\mathrm{R}x)\cdot 2) + 1 = \mathrm{R}(x\oplus \underline{1})$

26: $\neg\, \mathrm{Q}(x\oplus \underline{1}) = (\mathrm{Q}x)\cdot 2 \quad \vee \quad \mathrm{Half}(\mathrm{Q}(x\oplus \underline{1})) = \mathrm{Q}x$ by
8: $\mathrm{Half}((\mathrm{Q}x)\cdot 2) = \mathrm{Q}x$
16: $\neg\, \mathrm{Q}(x\oplus \underline{1}) = (\mathrm{Q}x)\cdot 2 \quad \vee \quad \mathrm{Half}(\mathrm{Q}(x\oplus \underline{1})) = \mathrm{Q}x \quad \vee \quad \neg\, \mathrm{Half}((\mathrm{Q}x)\cdot 2) = \mathrm{Q}x$

27: $\neg\, ((\mathrm{R}x)\cdot 2) + 1 = \mathrm{R}(x\oplus \underline{1}) \quad \vee \quad \mathrm{Half}(\mathrm{R}(x\oplus \underline{1})) = \mathrm{R}x$ by
9: $\mathrm{Half}(((\mathrm{R}x)\cdot 2) + 1) = \mathrm{R}x$
17: $\neg\, ((\mathrm{R}x)\cdot 2)+1 = \mathrm{R}(x\oplus \underline{1}) \quad \vee \quad \neg\, \mathrm{Half}(((\mathrm{R}x)\cdot 2)+1) = \mathrm{R}x \quad \vee \quad \mathrm{Half}(\mathrm{R}(x\oplus \underline{1})) = \mathrm{R}x$

28: $\mathrm{Q}(\mathrm{Chop}(x \oplus \underline{1})) = \mathrm{Q}x \quad \vee \quad \neg\, \mathrm{Half}(\mathrm{Q}(x \oplus \underline{1})) = \mathrm{Q}x$ by

19: $\mathrm{Q}(\mathrm{Chop}(x \oplus \underline{1})) = \mathrm{Half}(\mathrm{Q}(x \oplus \underline{1}))$

11: $\neg\, \mathrm{Q}(\mathrm{Chop}(x \oplus \underline{1})) = \mathrm{Half}(\mathrm{Q}(x \oplus \underline{1})) \quad \vee \quad \mathrm{Q}(\mathrm{Chop}(x \oplus \underline{1})) = \mathrm{Q}x \quad \vee \quad \neg\, \mathrm{Half}(\mathrm{Q}(x \oplus \underline{1})) = \mathrm{Q}x$

29: $\mathrm{R}(\mathrm{Chop}(x \oplus \underline{1})) = \mathrm{R}x \quad \vee \quad \neg\, \mathrm{Half}(\mathrm{R}(x \oplus \underline{1})) = \mathrm{R}x$ by

20: $\mathrm{R}(\mathrm{Chop}(x \oplus \underline{1})) = \mathrm{Half}(\mathrm{R}(x \oplus \underline{1}))$

12: $\neg\, \mathrm{R}(\mathrm{Chop}(x \oplus \underline{1})) = \mathrm{Half}(\mathrm{R}(x \oplus \underline{1})) \quad \vee \quad \mathrm{R}(\mathrm{Chop}(x \oplus \underline{1})) = \mathrm{R}x \quad \vee \quad \neg\, \mathrm{Half}(\mathrm{R}(x \oplus \underline{1})) = \mathrm{R}x$

30: $\mathrm{Half}(\mathrm{Q}(x \oplus \underline{1})) = \mathrm{Q}x$ by

23: $\mathrm{Q}(x \oplus \underline{1}) = (\mathrm{Q}x) \cdot 2$

26: $\neg\, \mathrm{Q}(x \oplus \underline{1}) = (\mathrm{Q}x) \cdot 2 \quad \vee \quad \mathrm{Half}(\mathrm{Q}(x \oplus \underline{1})) = \mathrm{Q}x$

31: $((\mathrm{R}x) \cdot 2) + 1 = \mathrm{R}(x \oplus \underline{1})$ by

24: $((\mathrm{R}x) \cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}(x \oplus \underline{1})$

25: $\neg\, ((\mathrm{R}x) \cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}(x \oplus \underline{1}) \quad \vee \quad ((\mathrm{R}x) \cdot 2) + 1 = \mathrm{R}(x \oplus \underline{1})$

32: $\mathrm{Q}(\mathrm{Chop}(x \oplus \underline{1})) = \mathrm{Q}x$ by

30: $\mathrm{Half}(\mathrm{Q}(x \oplus \underline{1})) = \mathrm{Q}x$

28: $\mathrm{Q}(\mathrm{Chop}(x \oplus \underline{1})) = \mathrm{Q}x \quad \vee \quad \neg\, \mathrm{Half}(\mathrm{Q}(x \oplus \underline{1})) = \mathrm{Q}x$

33: $\mathrm{Half}(\mathrm{R}(x \oplus \underline{1})) = \mathrm{R}x$ by

31: $((\mathrm{R}x) \cdot 2) + 1 = \mathrm{R}(x \oplus \underline{1})$

27: $\neg\, ((\mathrm{R}x) \cdot 2) + 1 = \mathrm{R}(x \oplus \underline{1}) \quad \vee \quad \mathrm{Half}(\mathrm{R}(x \oplus \underline{1})) = \mathrm{R}x$

34: $\neg\, \mathrm{R}(\mathrm{Chop}(x \oplus \underline{1})) = \mathrm{R}x$ by

32: $\mathrm{Q}(\mathrm{Chop}(x \oplus \underline{1})) = \mathrm{Q}x$

18: $\neg\, \mathrm{Q}(\mathrm{Chop}(x \oplus \underline{1})) = \mathrm{Q}x \quad \vee \quad \neg\, \mathrm{R}(\mathrm{Chop}(x \oplus \underline{1})) = \mathrm{R}x$

35: $\mathrm{R}(\mathrm{Chop}(x \oplus \underline{1})) = \mathrm{R}x$ by

33: $\mathrm{Half}(\mathrm{R}(x \oplus \underline{1})) = \mathrm{R}x$

29: $\mathrm{R}(\mathrm{Chop}(x \oplus \underline{1})) = \mathrm{R}x \quad \vee \quad \neg\, \mathrm{Half}(\mathrm{R}(x \oplus \underline{1})) = \mathrm{R}x$

36: QEA by

34: $\neg\, \mathrm{R}(\mathrm{Chop}(x \oplus \underline{1})) = \mathrm{R}x$

35: $\mathrm{R}(\mathrm{Chop}(x \oplus \underline{1})) = \mathrm{R}x$