Proof of Theorem 243

The theorem to be proved is

$\mathrm{Chop}(x \oplus \underline{0}) = x$ ⋆

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\, (\mathrm{Chop}(x \oplus \underline{0})) = (x)]]$

Special cases of the hypothesis and previous results:

0: $\neg\, \mathrm{Chop}(x \oplus \underline{0}) = x$ from H:x

1: $\neg\, x \oplus \underline{0} = \epsilon$ from 242;x

2: $x \oplus \underline{0} = \epsilon \quad \vee \quad \mathrm{Q}(\mathrm{Chop}(x \oplus \underline{0})) = \mathrm{Half}(\mathrm{Q}(x \oplus \underline{0}))$ from 239;$x \oplus \underline{0}$

3: $x \oplus \underline{0} = \epsilon \quad \vee \quad \mathrm{R}(\mathrm{Chop}(x \oplus \underline{0})) = \mathrm{Half}(\mathrm{R}(x \oplus \underline{0}))$ from 239;$x \oplus \underline{0}$

4: $(\mathrm{Q}x) \cdot (\mathrm{Q}\underline{0}) = \mathrm{Q}(x \oplus \underline{0})$ from 180;x;$\underline{0}$

5: $((\mathrm{R}x) \cdot (\mathrm{Q}\underline{0})) + (\mathrm{R}\underline{0}) = \mathrm{R}(x \oplus \underline{0})$ from 180;x;$\underline{0}$

6: $\mathrm{Q}\underline{0} = 2$ from 191

7: $\mathrm{R}\underline{0} = 0$ from 191

8: $\mathrm{Half}((\mathrm{Q}x) \cdot 2) = \mathrm{Q}x$ from 241;$\mathrm{Q}x$

9: $\mathrm{Half}((\mathrm{R}x) \cdot 2) = \mathrm{R}x$ from 241;$\mathrm{R}x$

10: $((\mathrm{R}x) \cdot 2) + 0 = (\mathrm{R}x) \cdot 2$ from 12;$(\mathrm{R}x) \cdot 2$

11: $\neg\, \mathrm{Q}(\mathrm{Chop}(x \oplus \underline{0})) = \mathrm{Q}x \quad \vee \quad \neg\, \mathrm{R}(\mathrm{Chop}(x \oplus \underline{0})) = \mathrm{R}x \quad \vee \quad \mathrm{Chop}(x \oplus \underline{0}) = x$ from 193;x;$\mathrm{Chop}(x \oplus \underline{0})$

Equality substitutions:

12: $\neg\, \mathrm{Q}(\mathrm{Chop}(x \oplus \underline{0})) = \mathrm{Half}(\mathrm{Q}(x \oplus \underline{0})) \quad \vee \quad \mathrm{Q}(\mathrm{Chop}(x \oplus \underline{0})) = \mathrm{Q}x \quad \vee \quad \neg\, \mathrm{Half}(\mathrm{Q}(x \oplus \underline{0})) = \mathrm{Q}x$

13: $\neg\, \mathrm{R}(\mathrm{Chop}(x \oplus \underline{0})) = \mathrm{Half}(\mathrm{R}(x \oplus \underline{0})) \quad \vee \quad \mathrm{R}(\mathrm{Chop}(x \oplus \underline{0})) = \mathrm{R}x \quad \vee \quad \neg\, \mathrm{Half}(\mathrm{R}(x \oplus \underline{0})) = \mathrm{R}x$

14: $\neg\, \mathrm{Q}\underline{0} = 2 \quad \vee \quad \neg\, (\mathrm{Q}x) \cdot (\mathrm{Q}\underline{0}) = \mathrm{Q}(x \oplus \underline{0}) \quad \vee \quad (\mathrm{Q}x) \cdot (2) = \mathrm{Q}(x \oplus \underline{0})$

15: $\neg\, \mathrm{Q}\underline{0} = 2 \quad \vee \quad ((\mathrm{R}x) \cdot (\mathrm{Q}\underline{0})) + 0 = (\mathrm{R}x) \cdot 2 \quad \vee \quad \neg\, ((\mathrm{R}x) \cdot (2)) + 0 = (\mathrm{R}x) \cdot 2$

16: $\neg\, \mathrm{R}\underline{0} = 0 \quad \vee \quad \neg\, ((\mathrm{R}x) \cdot (\mathrm{Q}\underline{0})) + (\mathrm{R}\underline{0}) = \mathrm{R}(x \oplus \underline{0}) \quad \vee \quad ((\mathrm{R}x) \cdot (\mathrm{Q}\underline{0})) + (0) = \mathrm{R}(x \oplus \underline{0})$

17: $\neg\, \mathrm{Q}(x \oplus \underline{0}) = (\mathrm{Q}x) \cdot 2 \quad \vee \quad \mathrm{Half}(\mathrm{Q}(x \oplus \underline{0})) = \mathrm{Q}x \quad \vee \quad \neg\, \mathrm{Half}((\mathrm{Q}x) \cdot 2) = \mathrm{Q}x$

18: $\neg\, ((\mathrm{R}x) \cdot (\mathrm{Q}\underline{0})) + 0 = \mathrm{R}(x \oplus \underline{0}) \quad \vee \quad \neg\, ((\mathrm{R}x) \cdot (\mathrm{Q}\underline{0})) + 0 = (\mathrm{R}x) \cdot 2 \quad \vee \quad \mathrm{R}(x \oplus \underline{0}) = (\mathrm{R}x) \cdot 2$

19: $\neg\, \mathrm{R}(x \oplus \underline{0}) = (\mathrm{R}x) \cdot 2 \quad \vee \quad \mathrm{Half}(\mathrm{R}(x \oplus \underline{0})) = \mathrm{R}x \quad \vee \quad \neg\, \mathrm{Half}((\mathrm{R}x) \cdot 2) = \mathrm{R}x$

Inferences:

20: $\neg\, \mathrm{Q}(\mathrm{Chop}(x \oplus \underline{0})) = \mathrm{Q}x \quad \vee \quad \neg\, \mathrm{R}(\mathrm{Chop}(x \oplus \underline{0})) = \mathrm{R}x$ by

0: $\neg\, \mathrm{Chop}(x \oplus \underline{0}) = x$

11: $\neg\, \mathrm{Q}(\mathrm{Chop}(x \oplus \underline{0})) = \mathrm{Q}x \quad \vee \quad \neg\, \mathrm{R}(\mathrm{Chop}(x \oplus \underline{0})) = \mathrm{R}x \quad \vee \quad \mathrm{Chop}(x \oplus \underline{0}) = x$

21: $\mathrm{Q}(\mathrm{Chop}(x \oplus \underline{0})) = \mathrm{Half}(\mathrm{Q}(x \oplus \underline{0}))$ by

1: $\neg\, x \oplus \underline{0} = \epsilon$

2: $x \oplus \underline{0} = \epsilon \quad \vee \quad \mathrm{Q}(\mathrm{Chop}(x \oplus \underline{0})) = \mathrm{Half}(\mathrm{Q}(x \oplus \underline{0}))$

22: $\mathrm{R}(\mathrm{Chop}(x \oplus \underline{0})) = \mathrm{Half}(\mathrm{R}(x \oplus \underline{0}))$ by

1: $\neg\, x \oplus \underline{0} = \epsilon$

3: $x \oplus \underline{0} = \epsilon \quad \vee \quad \mathrm{R}(\mathrm{Chop}(x \oplus \underline{0})) = \mathrm{Half}(\mathrm{R}(x \oplus \underline{0}))$

23: $\neg\, \mathrm{Q}\underline{0} = 2 \quad \vee \quad \mathrm{Q}(x \oplus \underline{0}) = (\mathrm{Q}x) \cdot 2$ by

4: $(\mathrm{Q}x) \cdot (\mathrm{Q}\underline{0}) = \mathrm{Q}(x \oplus \underline{0})$

14: $\neg\, \mathrm{Q}\underline{0} = 2 \quad \vee \quad \neg\, (\mathrm{Q}x) \cdot (\mathrm{Q}\underline{0}) = \mathrm{Q}(x \oplus \underline{0}) \quad \vee \quad \mathrm{Q}(x \oplus \underline{0}) = (\mathrm{Q}x) \cdot 2$

24: $\neg\, \mathrm{R}\underline{0} = 0 \quad \vee \quad ((\mathrm{R}x) \cdot (\mathrm{Q}\underline{0})) + 0 = \mathrm{R}(x \oplus \underline{0})$ by

5: $((\mathrm{R}x) \cdot (\mathrm{Q}\underline{0})) + (\mathrm{R}\underline{0}) = \mathrm{R}(x \oplus \underline{0})$

16: $\neg\, \mathrm{R}\underline{0} = 0 \quad \vee \quad \neg\, ((\mathrm{R}x) \cdot (\mathrm{Q}\underline{0})) + (\mathrm{R}\underline{0}) = \mathrm{R}(x \oplus \underline{0}) \quad \vee \quad ((\mathrm{R}x) \cdot (\mathrm{Q}\underline{0})) + 0 = \mathrm{R}(x \oplus \underline{0})$

25: $((\mathrm{R}x) \cdot (\mathrm{Q}\underline{0})) + 0 = (\mathrm{R}x) \cdot 2 \quad \vee \quad \neg\, ((\mathrm{R}x) \cdot 2) + 0 = (\mathrm{R}x) \cdot 2$ by

6: $\mathrm{Q}\underline{0} = 2$

15: $\neg\, \mathrm{Q}\underline{0} = 2 \quad \vee \quad ((\mathrm{R}x) \cdot (\mathrm{Q}\underline{0})) + 0 = (\mathrm{R}x) \cdot 2 \quad \vee \quad \neg\, ((\mathrm{R}x) \cdot 2) + 0 = (\mathrm{R}x) \cdot 2$

26: $\mathrm{Q}(x \oplus \underline{0}) = (\mathrm{Q}x) \cdot 2$ by

6: $\mathrm{Q}\underline{0} = 2$

23: $\neg\, \mathrm{Q}\underline{0} = 2 \quad \vee \quad \mathrm{Q}(x \oplus \underline{0}) = (\mathrm{Q}x) \cdot 2$

27: $((\mathrm{R}x) \cdot (\mathrm{Q}\underline{0})) + 0 = \mathrm{R}(x \oplus \underline{0})$ by

7: $\mathrm{R}\underline{0} = 0$

24: $\neg\, \mathrm{R}\underline{0} = 0 \quad \vee \quad ((\mathrm{R}x) \cdot (\mathrm{Q}\underline{0})) + 0 = \mathrm{R}(x \oplus \underline{0})$

28: $\neg\, \mathrm{Q}(x \oplus \underline{0}) = (\mathrm{Q}x) \cdot 2 \quad \vee \quad \mathrm{Half}(\mathrm{Q}(x \oplus \underline{0})) = \mathrm{Q}x$ by

8: $\mathrm{Half}((\mathrm{Q}x) \cdot 2) = \mathrm{Q}x$

17: $\neg\, \mathrm{Q}(x \oplus \underline{0}) = (\mathrm{Q}x) \cdot 2 \quad \vee \quad \mathrm{Half}(\mathrm{Q}(x \oplus \underline{0})) = \mathrm{Q}x \quad \vee \quad \neg\, \mathrm{Half}((\mathrm{Q}x) \cdot 2) = \mathrm{Q}x$

29: $\neg\, \mathrm{R}(x \oplus \underline{0}) = (\mathrm{R}x)\cdot 2 \quad \vee \quad \mathrm{Half}(\mathrm{R}(x \oplus \underline{0})) = \mathrm{R}x$ by
9: $\mathrm{Half}((\mathrm{R}x)\cdot 2) = \mathrm{R}x$
19: $\neg\, \mathrm{R}(x \oplus \underline{0}) = (\mathrm{R}x)\cdot 2 \quad \vee \quad \mathrm{Half}(\mathrm{R}(x \oplus \underline{0})) = \mathrm{R}x \quad \vee \quad \neg\, \mathrm{Half}((\mathrm{R}x)\cdot 2) = \mathrm{R}x$

30: $((\mathrm{R}x)\cdot(\mathrm{Q}\underline{0})) + 0 = (\mathrm{R}x)\cdot 2$ by
10: $((\mathrm{R}x)\cdot 2) + 0 = (\mathrm{R}x)\cdot 2$
25: $((\mathrm{R}x)\cdot(\mathrm{Q}\underline{0})) + 0 = (\mathrm{R}x)\cdot 2 \quad \vee \quad \neg\, ((\mathrm{R}x)\cdot 2) + 0 = (\mathrm{R}x)\cdot 2$

31: $\mathrm{Q}(\mathrm{Chop}(x \oplus \underline{0})) = \mathrm{Q}x \quad \vee \quad \neg\, \mathrm{Half}(\mathrm{Q}(x \oplus \underline{0})) = \mathrm{Q}x$ by
21: $\mathrm{Q}(\mathrm{Chop}(x \oplus \underline{0})) = \mathrm{Half}(\mathrm{Q}(x \oplus \underline{0}))$
12: $\neg\, \mathrm{Q}(\mathrm{Chop}(x \oplus \underline{0})) = \mathrm{Half}(\mathrm{Q}(x \oplus \underline{0})) \quad \vee \quad \mathrm{Q}(\mathrm{Chop}(x \oplus \underline{0})) = \mathrm{Q}x \quad \vee$ $\neg\, \mathrm{Half}(\mathrm{Q}(x \oplus \underline{0})) = \mathrm{Q}x$

32: $\mathrm{R}(\mathrm{Chop}(x \oplus \underline{0})) = \mathrm{R}x \quad \vee \quad \neg\, \mathrm{Half}(\mathrm{R}(x \oplus \underline{0})) = \mathrm{R}x$ by
22: $\mathrm{R}(\mathrm{Chop}(x \oplus \underline{0})) = \mathrm{Half}(\mathrm{R}(x \oplus \underline{0}))$
13: $\neg\, \mathrm{R}(\mathrm{Chop}(x \oplus \underline{0})) = \mathrm{Half}(\mathrm{R}(x \oplus \underline{0})) \quad \vee \quad \mathrm{R}(\mathrm{Chop}(x \oplus \underline{0})) = \mathrm{R}x \quad \vee \quad \neg\, \mathrm{Half}(\mathrm{R}(x \oplus$ $\underline{0})) = \mathrm{R}x$

33: $\mathrm{Half}(\mathrm{Q}(x \oplus \underline{0})) = \mathrm{Q}x$ by
26: $\mathrm{Q}(x \oplus \underline{0}) = (\mathrm{Q}x)\cdot 2$
28: $\neg\, \mathrm{Q}(x \oplus \underline{0}) = (\mathrm{Q}x)\cdot 2 \quad \vee \quad \mathrm{Half}(\mathrm{Q}(x \oplus \underline{0})) = \mathrm{Q}x$

34: $\neg\, ((\mathrm{R}x)\cdot(\mathrm{Q}\underline{0})) + 0 = (\mathrm{R}x)\cdot 2 \quad \vee \quad \mathrm{R}(x \oplus \underline{0}) = (\mathrm{R}x)\cdot 2$ by
27: $((\mathrm{R}x)\cdot(\mathrm{Q}\underline{0})) + 0 = \mathrm{R}(x \oplus \underline{0})$
18: $\neg\, ((\mathrm{R}x)\cdot(\mathrm{Q}\underline{0})) + 0 = \mathrm{R}(x \oplus \underline{0}) \quad \vee \quad \neg\, ((\mathrm{R}x)\cdot(\mathrm{Q}\underline{0})) + 0 = (\mathrm{R}x)\cdot 2 \quad \vee \quad \mathrm{R}(x \oplus \underline{0}) =$ $(\mathrm{R}x)\cdot 2$

35: $\mathrm{R}(x \oplus \underline{0}) = (\mathrm{R}x)\cdot 2$ by
30: $((\mathrm{R}x)\cdot(\mathrm{Q}\underline{0})) + 0 = (\mathrm{R}x)\cdot 2$
34: $\neg\, ((\mathrm{R}x)\cdot(\mathrm{Q}\underline{0})) + 0 = (\mathrm{R}x)\cdot 2 \quad \vee \quad \mathrm{R}(x \oplus \underline{0}) = (\mathrm{R}x)\cdot 2$

36: $\mathrm{Q}(\mathrm{Chop}(x \oplus \underline{0})) = \mathrm{Q}x$ by
33: $\mathrm{Half}(\mathrm{Q}(x \oplus \underline{0})) = \mathrm{Q}x$
31: $\mathrm{Q}(\mathrm{Chop}(x \oplus \underline{0})) = \mathrm{Q}x \quad \vee \quad \neg\, \mathrm{Half}(\mathrm{Q}(x \oplus \underline{0})) = \mathrm{Q}x$

37: $\mathrm{Half}(\mathrm{R}(x \oplus \underline{0})) = \mathrm{R}x$ by
35: $\mathrm{R}(x \oplus \underline{0}) = (\mathrm{R}x)\cdot 2$
29: $\neg\, \mathrm{R}(x \oplus \underline{0}) = (\mathrm{R}x)\cdot 2 \quad \vee \quad \mathrm{Half}(\mathrm{R}(x \oplus \underline{0})) = \mathrm{R}x$

38: $\neg\, \mathrm{R}(\mathrm{Chop}(x \oplus \underline{0})) = \mathrm{R}x$ by
36: $\mathrm{Q}(\mathrm{Chop}(x \oplus \underline{0})) = \mathrm{Q}x$
20: $\neg\, \mathrm{Q}(\mathrm{Chop}(x \oplus \underline{0})) = \mathrm{Q}x \quad \vee \quad \neg\, \mathrm{R}(\mathrm{Chop}(x \oplus \underline{0})) = \mathrm{R}x$

39: $\mathrm{R}(\mathrm{Chop}(x \oplus \underline{0})) = \mathrm{R}x$ by

37: $\mathrm{Half}(\mathrm{R}(x \oplus \underline{0})) = \mathrm{R}x$

32: $\mathrm{R}(\mathrm{Chop}(x \oplus \underline{0})) = \mathrm{R}x \quad \vee \quad \neg\, \mathrm{Half}(\mathrm{R}(x \oplus \underline{0})) = \mathrm{R}x$

40: QEA by

38: $\neg\, \mathrm{R}(\mathrm{Chop}(x \oplus \underline{0})) = \mathrm{R}x$

39: $\mathrm{R}(\mathrm{Chop}(x \oplus \underline{0})) = \mathrm{R}x$