Proof of Theorem 242

The theorem to be proved is

$x \oplus \underline{0} \neq \epsilon \quad \& \quad x \oplus \underline{1} \neq \epsilon$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x \oplus \underline{0}) = (\epsilon) \quad \vee \quad (x \oplus \underline{1}) = (\epsilon)]]$

Special cases of the hypothesis and previous results:

0: $x \oplus \underline{0} = \epsilon \quad \vee \quad x \oplus \underline{1} = \epsilon$ from H:x

1: $\neg\, x \oplus \underline{0} = \epsilon \quad \vee \quad \underline{0} = \epsilon$ from 204;x;$\underline{0}$

2: $\neg\, x \oplus \underline{1} = \epsilon \quad \vee \quad \underline{1} = \epsilon$ from 204;x;$\underline{1}$

3: $\neg\, \underline{0} = \epsilon$ from 188

4: $\neg\, \underline{1} = \epsilon$ from 188

Inferences:

5: $\neg\, x \oplus \underline{0} = \epsilon$ by
3: $\neg\, \underline{0} = \epsilon$
1: $\neg\, x \oplus \underline{0} = \epsilon \quad \vee \quad \underline{0} = \epsilon$

6: $\neg\, x \oplus \underline{1} = \epsilon$ by
4: $\neg\, \underline{1} = \epsilon$
2: $\neg\, x \oplus \underline{1} = \epsilon \quad \vee \quad \underline{1} = \epsilon$

7: $x \oplus \underline{1} = \epsilon$ by
5: $\neg\, x \oplus \underline{0} = \epsilon$
0: $x \oplus \underline{0} = \epsilon \quad \vee \quad x \oplus \underline{1} = \epsilon$

8: QEA by
6: $\neg\, x \oplus \underline{1} = \epsilon$
7: $x \oplus \underline{1} = \epsilon$