

Proof of Theorem 241b

The theorem to be proved is

$$\text{Half}(0 \cdot 2) = 0 \quad \& \quad \text{Half}(0 \cdot 2 + 1) = 0$$

Suppose the theorem does not hold. Then, with the variables held fixed,

$$(H) \quad [[\neg (\text{Half}(0 \cdot 2)) = 0) \vee \neg (\text{Half}((0 \cdot 2) + 1)) = 0]]$$

Special cases of the hypothesis and previous results:

$$0: \quad \neg \text{Half}(0 \cdot 2) = 0 \quad \vee \quad \neg \text{Half}((0 \cdot 2) + 1) = 0 \quad \text{from } H$$

$$1: \quad 0 \cdot 2 = 0 \quad \text{from } \text{\color{blue}103};2$$

$$2: \quad 0 + 1 = 1 \quad \text{from } \text{\color{blue}97};1$$

$$3: \quad \text{Half}0 = 0 \quad \text{from } \text{\color{blue}221}$$

$$4: \quad \text{Half}1 = 0 \quad \text{from } \text{\color{blue}221}$$

Equality substitutions:

$$5: \quad \neg 0 \cdot 2 = 0 \quad \vee \quad \text{Half}(0 \cdot 2) = 0 \quad \vee \quad \neg \text{Half}(0) = 0$$

$$6: \quad \neg 0 \cdot 2 = 0 \quad \vee \quad \text{Half}((0 \cdot 2) + 1) = 0 \quad \vee \quad \neg \text{Half}((0) + 1) = 0$$

$$7: \quad \neg 0 + 1 = 1 \quad \vee \quad \text{Half}(0 + 1) = 0 \quad \vee \quad \neg \text{Half}(1) = 0$$

Inferences:

$$8: \quad \text{Half}(0 \cdot 2) = 0 \quad \vee \quad \neg \text{Half}0 = 0 \quad \text{by}$$

$$1: \quad 0 \cdot 2 = 0$$

$$5: \quad \neg 0 \cdot 2 = 0 \quad \vee \quad \text{Half}(0 \cdot 2) = 0 \quad \vee \quad \neg \text{Half}0 = 0$$

$$9: \quad \text{Half}((0 \cdot 2) + 1) = 0 \quad \vee \quad \neg \text{Half}(0 + 1) = 0 \quad \text{by}$$

$$1: \quad 0 \cdot 2 = 0$$

$$6: \quad \neg 0 \cdot 2 = 0 \quad \vee \quad \text{Half}((0 \cdot 2) + 1) = 0 \quad \vee \quad \neg \text{Half}(0 + 1) = 0$$

$$10: \quad \text{Half}(0 + 1) = 0 \quad \vee \quad \neg \text{Half}1 = 0 \quad \text{by}$$

$$2: \quad 0 + 1 = 1$$

$$7: \quad \neg 0 + 1 = 1 \quad \vee \quad \text{Half}(0 + 1) = 0 \quad \vee \quad \neg \text{Half}1 = 0$$

- 11: $\text{Half}(0 \cdot 2) = 0$ by
 3: $\text{Half}0 = 0$
 8: $\text{Half}(0 \cdot 2) = 0 \vee \neg \text{Half}0 = 0$
- 12: $\text{Half}(0 + 1) = 0$ by
 4: $\text{Half}1 = 0$
 10: $\text{Half}(0 + 1) = 0 \vee \neg \text{Half}1 = 0$
- 13: $\neg \text{Half}((0 \cdot 2) + 1) = 0$ by
 11: $\text{Half}(0 \cdot 2) = 0$
 0: $\neg \text{Half}(0 \cdot 2) = 0 \vee \neg \text{Half}((0 \cdot 2) + 1) = 0$
- 14: $\text{Half}((0 \cdot 2) + 1) = 0$ by
 12: $\text{Half}(0 + 1) = 0$
 9: $\text{Half}((0 \cdot 2) + 1) = 0 \vee \neg \text{Half}(0 + 1) = 0$
- 15: *QEA* by
 13: $\neg \text{Half}((0 \cdot 2) + 1) = 0$
 14: $\text{Half}((0 \cdot 2) + 1) = 0$