Proof of Theorem 240

The theorem to be proved is

$\text{Chop}\,\underline{0} = \epsilon \quad \& \quad \text{Chop}\,\underline{1} = \epsilon$ ⋆

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\,(\text{Chop}\underline{0}) = (\epsilon) \quad \vee \quad \neg\,(\text{Chop}\underline{1}) = (\epsilon)]]$

Special cases of the hypothesis and previous results:

0: $\neg\,\text{Chop}\underline{0} = \epsilon \quad \vee \quad \neg\,\text{Chop}\underline{1} = \epsilon$ from H

1: $\underline{0} = \epsilon \quad \vee \quad \text{Q}(\text{Chop}\underline{0}) = \text{Half}(\text{Q}\underline{0})$ from 239;$\underline{0}$

2: $\underline{1} = \epsilon \quad \vee \quad \text{Q}(\text{Chop}\underline{1}) = \text{Half}(\text{Q}\underline{1})$ from 239;$\underline{1}$

3: $\neg\,\underline{0} = \epsilon$ from 188

4: $\neg\,\underline{1} = \epsilon$ from 188

5: $\text{Q}\underline{0} = 2$ from 191

6: $\text{Q}\underline{1} = 2$ from 192

7: $\text{Half2} = 1$ from 221

8: $\neg\,\text{Q}(\text{Chop}\underline{0}) = 1 \quad \vee \quad \text{Chop}\underline{0} = \epsilon$ from 203;Chop$\underline{0}$

9: $\neg\,\text{Q}(\text{Chop}\underline{1}) = 1 \quad \vee \quad \text{Chop}\underline{1} = \epsilon$ from 203;Chop$\underline{1}$

Equality substitutions:

10: $\neg\,\text{Q}(\text{Chop}\underline{0}) = \text{Half}(\text{Q}\underline{0}) \quad \vee \quad \text{Q}(\text{Chop}\underline{0}) = 1 \quad \vee \quad \neg\,\text{Half}(\text{Q}\underline{0}) = 1$

11: $\neg\,\text{Q}(\text{Chop}\underline{1}) = \text{Half}(\text{Q}\underline{1}) \quad \vee \quad \text{Q}(\text{Chop}\underline{1}) = 1 \quad \vee \quad \neg\,\text{Half}(\text{Q}\underline{1}) = 1$

12: $\neg\,\text{Q}\underline{0} = 2 \quad \vee \quad \text{Half}(\text{Q}\underline{0}) = 1 \quad \vee \quad \neg\,\text{Half}(2) = 1$

13: $\neg\,\text{Q}\underline{1} = 2 \quad \vee \quad \text{Half}(\text{Q}\underline{1}) = 1 \quad \vee \quad \neg\,\text{Half}(2) = 1$

Inferences:

14: $\text{Q}(\text{Chop}\underline{0}) = \text{Half}(\text{Q}\underline{0})$ by

3: $\neg\,\underline{0} = \epsilon$

1: $\underline{0} = \epsilon \quad \vee \quad \text{Q}(\text{Chop}\underline{0}) = \text{Half}(\text{Q}\underline{0})$

15: $\mathrm{Q}(\mathrm{Chop}\underline{1}) = \mathrm{Half}(\mathrm{Q}\underline{1})$ by

4: $\neg\ \underline{1} = \epsilon$

2: $\underline{1} = \epsilon \quad \vee \quad \mathrm{Q}(\mathrm{Chop}\underline{1}) = \mathrm{Half}(\mathrm{Q}\underline{1})$

16: $\mathrm{Half}(\mathrm{Q}\underline{0}) = 1 \quad \vee \quad \neg\ \mathrm{Half}2 = 1$ by

5: $\mathrm{Q}\underline{0} = 2$

12: $\neg\ \mathrm{Q}\underline{0} = 2 \quad \vee \quad \mathrm{Half}(\mathrm{Q}\underline{0}) = 1 \quad \vee \quad \neg\ \mathrm{Half}2 = 1$

17: $\mathrm{Half}(\mathrm{Q}\underline{1}) = 1 \quad \vee \quad \neg\ \mathrm{Half}2 = 1$ by

6: $\mathrm{Q}\underline{1} = 2$

13: $\neg\ \mathrm{Q}\underline{1} = 2 \quad \vee \quad \mathrm{Half}(\mathrm{Q}\underline{1}) = 1 \quad \vee \quad \neg\ \mathrm{Half}2 = 1$

18: $\mathrm{Half}(\mathrm{Q}\underline{0}) = 1$ by

7: $\mathrm{Half}2 = 1$

16: $\mathrm{Half}(\mathrm{Q}\underline{0}) = 1 \quad \vee \quad \neg\ \mathrm{Half}2 = 1$

19: $\mathrm{Half}(\mathrm{Q}\underline{1}) = 1$ by

7: $\mathrm{Half}2 = 1$

17: $\mathrm{Half}(\mathrm{Q}\underline{1}) = 1 \quad \vee \quad \neg\ \mathrm{Half}2 = 1$

20: $\mathrm{Q}(\mathrm{Chop}\underline{0}) = 1 \quad \vee \quad \neg\ \mathrm{Half}(\mathrm{Q}\underline{0}) = 1$ by

14: $\mathrm{Q}(\mathrm{Chop}\underline{0}) = \mathrm{Half}(\mathrm{Q}\underline{0})$

10: $\neg\ \mathrm{Q}(\mathrm{Chop}\underline{0}) = \mathrm{Half}(\mathrm{Q}\underline{0}) \quad \vee \quad \mathrm{Q}(\mathrm{Chop}\underline{0}) = 1 \quad \vee \quad \neg\ \mathrm{Half}(\mathrm{Q}\underline{0}) = 1$

21: $\mathrm{Q}(\mathrm{Chop}\underline{1}) = 1 \quad \vee \quad \neg\ \mathrm{Half}(\mathrm{Q}\underline{1}) = 1$ by

15: $\mathrm{Q}(\mathrm{Chop}\underline{1}) = \mathrm{Half}(\mathrm{Q}\underline{1})$

11: $\neg\ \mathrm{Q}(\mathrm{Chop}\underline{1}) = \mathrm{Half}(\mathrm{Q}\underline{1}) \quad \vee \quad \mathrm{Q}(\mathrm{Chop}\underline{1}) = 1 \quad \vee \quad \neg\ \mathrm{Half}(\mathrm{Q}\underline{1}) = 1$

22: $\mathrm{Q}(\mathrm{Chop}\underline{0}) = 1$ by

18: $\mathrm{Half}(\mathrm{Q}\underline{0}) = 1$

20: $\mathrm{Q}(\mathrm{Chop}\underline{0}) = 1 \quad \vee \quad \neg\ \mathrm{Half}(\mathrm{Q}\underline{0}) = 1$

23: $\mathrm{Q}(\mathrm{Chop}\underline{1}) = 1$ by

19: $\mathrm{Half}(\mathrm{Q}\underline{1}) = 1$

21: $\mathrm{Q}(\mathrm{Chop}\underline{1}) = 1 \quad \vee \quad \neg\ \mathrm{Half}(\mathrm{Q}\underline{1}) = 1$

24: $\mathrm{Chop}\underline{0} = \epsilon$ by

22: $\mathrm{Q}(\mathrm{Chop}\underline{0}) = 1$

8: $\neg\ \mathrm{Q}(\mathrm{Chop}\underline{0}) = 1 \quad \vee \quad \mathrm{Chop}\underline{0} = \epsilon$

25: $\mathrm{Chop}\underline{1} = \epsilon$ by

23: $\mathrm{Q}(\mathrm{Chop}\underline{1}) = 1$

9: $\neg\ \mathrm{Q}(\mathrm{Chop}\underline{1}) = 1 \quad \vee \quad \mathrm{Chop}\underline{1} = \epsilon$

26: $\neg$ Chop$\underline{1} = \epsilon$ by

24: Chop$\underline{0} = \epsilon$

0: $\neg$ Chop$\underline{0} = \epsilon \quad \vee \quad \neg$ Chop$\underline{1} = \epsilon$

27: QEA by

25: Chop$\underline{1} = \epsilon$

26: $\neg$ Chop$\underline{1} = \epsilon$