## Proof of Theorem 24

The theorem to be proved is

$y - x = S0 \quad \rightarrow \quad Sx = y$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(y - x) = (S0)] \quad \& \quad [\neg\, (Sx) = (y)]]$

### Special cases of the hypothesis and previous results:

0: $y - x = S0$ from H:$y$:$x$

1: $\neg\, Sx = y$ from H:$y$:$x$

2: $\neg\, S0 = 0$ from 3;0

3: $y - x = 0 \ \vee \ x + (y - x) = y$ from 23;$y$;$x$

4: $x + 0 = x$ from 12;$x$;0

5: $S(x + 0) = x + (S0)$ from 12;$x$;0

### Equality substitutions:

6: $\neg\, y - x = S0 \ \vee \ \neg\, y - x = 0 \ \vee \ S0 = 0$

7: $\neg\, y - x = S0 \ \vee \ \neg\, x + (y - x) = y \ \vee \ x + (S0) = y$

8: $\neg\, x + 0 = x \ \vee \ \neg\, S(x + 0) = y \ \vee \ S(x) = y$

9: $\neg\, S(x + 0) = x + (S0) \ \vee \ S(x + 0) = y \ \vee \ \neg\, x + (S0) = y$

### Inferences:

10: $\neg\, y - x = 0 \ \vee \ S0 = 0$ by

0: $y - x = S0$

6: $\neg\, y - x = S0 \ \vee \ \neg\, y - x = 0 \ \vee \ S0 = 0$

11: $\neg\, x + (y - x) = y \ \vee \ x + (S0) = y$ by

0: $y - x = S0$

7: $\neg\, y - x = S0 \ \vee \ \neg\, x + (y - x) = y \ \vee \ x + (S0) = y$

12: $\neg\, x + 0 = x \ \vee \ \neg\, S(x + 0) = y$ by

1: $\neg\, Sx = y$

8: $\neg\, x + 0 = x \ \vee \ \neg\, S(x + 0) = y \ \vee \ Sx = y$

13: $\neg\ y - x = 0$ by

2: $\neg\ \mathrm{S}0 = 0$

10: $\neg\ y - x = 0 \quad \vee \quad \mathrm{S}0 = 0$

14: $\neg\ \mathrm{S}(x + 0) = y$ by

4: $x + 0 = x$

12: $\neg\ x + 0 = x \quad \vee \quad \neg\ \mathrm{S}(x + 0) = y$

15: $\mathrm{S}(x + 0) = y \quad \vee \quad \neg\ x + (\mathrm{S}0) = y$ by

5: $\mathrm{S}(x + 0) = x + (\mathrm{S}0)$

9: $\neg\ \mathrm{S}(x + 0) = x + (\mathrm{S}0) \quad \vee \quad \mathrm{S}(x + 0) = y \quad \vee \quad \neg\ x + (\mathrm{S}0) = y$

16: $x + (y - x) = y$ by

13: $\neg\ y - x = 0$

3: $y - x = 0 \quad \vee \quad x + (y - x) = y$

17: $\neg\ x + (\mathrm{S}0) = y$ by

14: $\neg\ \mathrm{S}(x + 0) = y$

15: $\mathrm{S}(x + 0) = y \quad \vee \quad \neg\ x + (\mathrm{S}0) = y$

18: $x + (\mathrm{S}0) = y$ by

16: $x + (y - x) = y$

11: $\neg\ x + (y - x) = y \quad \vee \quad x + (\mathrm{S}0) = y$

19: $QEA$ by

17: $\neg\ x + (\mathrm{S}0) = y$

18: $x + (\mathrm{S}0) = y$