Proof of Theorem 23b

The theorem to be proved is

$y - 0 \neq 0 \quad \rightarrow \quad 0 + (y - 0) = y$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\,(y - 0) = (0)] \quad \& \quad [\neg\,(0 + (y - 0)) = (y)]]$

Special cases of the hypothesis and previous results:

0: $\neg\,0 + (y - 0) = y$ from H:y

1: $y - 0 = y$ from 17;y

2: $0 + y = y + 0$ from 13;y

3: $y + 0 = y$ from 12;y

Equality substitutions:

4: $\neg\,y - 0 = y \quad \vee \quad 0 + (y - 0) = y \quad \vee \quad \neg\,0 + (y) = y$

5: $\neg\,0 + y = y + 0 \quad \vee \quad 0 + y = y \quad \vee \quad \neg\,y + 0 = y$

Inferences:

6: $\neg\,y - 0 = y \quad \vee \quad \neg\,0 + y = y$ by
0: $\neg\,0 + (y - 0) = y$
4: $\neg\,y - 0 = y \quad \vee \quad 0 + (y - 0) = y \quad \vee \quad \neg\,0 + y = y$

7: $\neg\,0 + y = y$ by
1: $y - 0 = y$
6: $\neg\,y - 0 = y \quad \vee \quad \neg\,0 + y = y$

8: $0 + y = y \quad \vee \quad \neg\,y + 0 = y$ by
2: $0 + y = y + 0$
5: $\neg\,0 + y = y + 0 \quad \vee \quad 0 + y = y \quad \vee \quad \neg\,y + 0 = y$

9: $0 + y = y$ by
3: $y + 0 = y$
8: $0 + y = y \quad \vee \quad \neg\,y + 0 = y$

10: QEA by
7: $\neg\,0 + y = y$
9: $0 + y = y$