Proof of Theorem 238

The theorem to be proved is

$x \neq \epsilon \quad \rightarrow \quad$ Half $\mathrm{Q}x$ is a power of two

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $\quad [[\neg\, (x) = (\epsilon)] \quad \& \quad [\neg\, (\mathrm{Half}(\mathrm{Q}x)) \text{ is a power of two}]]$

Special cases of the hypothesis and previous results:

0: $\neg\, \epsilon = x$ from H:x

1: $\neg$ Half$(\mathrm{Q}x)$ is a power of two from H:x

2: $\mathrm{Q}x$ is a power of two from 158;x

3: $\neg\, \mathrm{Q}x$ is a power of two $\vee\ 2 \uparrow y = \mathrm{Q}x$ from $129^{\rightarrow}$;$\mathrm{Q}x$:y

4: $2 \uparrow 0 = 1$ from 126;2

5: $\neg\, \mathrm{Q}x = 1 \ \vee\ \epsilon = x$ from 203;x

6: $0 = y \ \vee$ Half$(2 \uparrow y)$ is a power of two from 237;y

Equality substitutions:

7: $\neg\, 2 \uparrow y = \mathrm{Q}x \ \vee\ \neg$ Half$(2 \uparrow y)$ is a power of two $\vee$ Half$(\mathrm{Q}x)$ is a power of two

8: $\neg\, 2 \uparrow y = \mathrm{Q}x \ \vee\ \neg\, 2 \uparrow y = 1 \ \vee\ \mathrm{Q}x = 1$

9: $\neg\, 0 = y \ \vee\ \neg\, 2 \uparrow 0 = 1 \ \vee\ 2 \uparrow y = 1$

Inferences:

10: $\neg\, \mathrm{Q}x = 1$ by

0: $\neg\, \epsilon = x$

5: $\neg\, \mathrm{Q}x = 1 \ \vee\ \epsilon = x$

11: $\neg\, 2 \uparrow y = \mathrm{Q}x \ \vee\ \neg$ Half$(2 \uparrow y)$ is a power of two by

1: $\neg$ Half$(\mathrm{Q}x)$ is a power of two

7: $\neg\, 2 \uparrow y = \mathrm{Q}x \ \vee\ \neg$ Half$(2 \uparrow y)$ is a power of two $\vee$ Half$(\mathrm{Q}x)$ is a power of two

12: $2 \uparrow y = \mathrm{Q}x$ by

2: $\mathrm{Q}x$ is a power of two

3: $\neg\, \mathrm{Q}x$ is a power of two $\vee\ 2 \uparrow y = \mathrm{Q}x$

13: $\neg\, 0 = y \quad \vee \quad 2 \uparrow y = 1$ by

4: $2 \uparrow 0 = 1$

9: $\neg\, 0 = y \quad \vee \quad \neg\, 2 \uparrow 0 = 1 \quad \vee \quad 2 \uparrow y = 1$

14: $\neg\, 2 \uparrow y = \mathrm{Q}x \quad \vee \quad \neg\, 2 \uparrow y = 1$ by

10: $\neg\, \mathrm{Q}x = 1$

8: $\neg\, 2 \uparrow y = \mathrm{Q}x \quad \vee \quad \neg\, 2 \uparrow y = 1 \quad \vee \quad \mathrm{Q}x = 1$

15: $\neg$ Half$(2 \uparrow y)$ is a power of two by

12: $2 \uparrow y = \mathrm{Q}x$

11: $\neg\, 2 \uparrow y = \mathrm{Q}x \quad \vee \quad \neg$ Half$(2 \uparrow y)$ is a power of two

16: $\neg\, 2 \uparrow y = 1$ by

12: $2 \uparrow y = \mathrm{Q}x$

14: $\neg\, 2 \uparrow y = \mathrm{Q}x \quad \vee \quad \neg\, 2 \uparrow y = 1$

17: $0 = y$ by

15: $\neg$ Half$(2 \uparrow y)$ is a power of two

6: $0 = y \quad \vee$ Half$(2 \uparrow y)$ is a power of two

18: $\neg\, 0 = y$ by

16: $\neg\, 2 \uparrow y = 1$

13: $\neg\, 0 = y \quad \vee \quad 2 \uparrow y = 1$

19: QEA by

17: $0 = y$

18: $\neg\, 0 = y$