Proof of Theorem 237

The theorem to be proved is

$x \neq 0 \quad \rightarrow \quad \text{Half}(2 \uparrow x)$ is a power of two

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\,(x) = (0)] \quad \& \quad [\neg\,(\text{Half}(2 \uparrow x))$ is a power of two$]]$

Special cases of the hypothesis and previous results:

0: $\neg\, 0 = x$ from H;x

1: $\neg$ Half$(2 \uparrow x)$ is a power of two from H;x

2: $0 = x \quad \vee \quad \text{S}(\text{P}x) = x$ from 22;x

3: $\text{Half}(2 \uparrow (\text{S}(\text{P}x))) = 2 \uparrow (\text{P}x)$ from 235;Px

4: $2 \uparrow (\text{P}x)$ is a power of two from 131;Px

Equality substitutions:

5: $\neg\, \text{S}(\text{P}x) = x \quad \vee \quad \neg\, \text{Half}(2 \uparrow (\text{S}(\text{P}x))) = 2 \uparrow (\text{P}x) \quad \vee \quad \text{Half}(2 \uparrow (x)) = 2 \uparrow (\text{P}x)$

6: $\neg\, \text{Half}(2 \uparrow x) = 2 \uparrow (\text{P}x) \quad \vee \quad \text{Half}(2 \uparrow x)$ is a power of two $\quad \vee \quad \neg\, 2 \uparrow (\text{P}x)$ is a power of two

Inferences:

7: $\text{S}(\text{P}x) = x$ by

0: $\neg\, 0 = x$

2: $0 = x \quad \vee \quad \text{S}(\text{P}x) = x$

8: $\neg\, \text{Half}(2 \uparrow x) = 2 \uparrow (\text{P}x) \quad \vee \quad \neg\, 2 \uparrow (\text{P}x)$ is a power of two by

1: $\neg$ Half$(2 \uparrow x)$ is a power of two

6: $\neg\, \text{Half}(2 \uparrow x) = 2 \uparrow (\text{P}x) \quad \vee \quad \text{Half}(2 \uparrow x)$ is a power of two $\quad \vee \quad \neg\, 2 \uparrow (\text{P}x)$ is a power of two

9: $\neg\, \text{S}(\text{P}x) = x \quad \vee \quad \text{Half}(2 \uparrow x) = 2 \uparrow (\text{P}x)$ by

3: $\text{Half}(2 \uparrow (\text{S}(\text{P}x))) = 2 \uparrow (\text{P}x)$

5: $\neg\, \text{S}(\text{P}x) = x \quad \vee \quad \neg\, \text{Half}(2 \uparrow (\text{S}(\text{P}x))) = 2 \uparrow (\text{P}x) \quad \vee \quad \text{Half}(2 \uparrow x) = 2 \uparrow (\text{P}x)$

10: $\neg\ \mathrm{Half}(2 \uparrow x) = 2 \uparrow (\mathrm{P}x)$ by

4: $2 \uparrow (\mathrm{P}x)$ is a power of two

8: $\neg\ \mathrm{Half}(2 \uparrow x) = 2 \uparrow (\mathrm{P}x) \quad \vee \quad \neg\ 2 \uparrow (\mathrm{P}x)$ is a power of two

11: $\mathrm{Half}(2 \uparrow x) = 2 \uparrow (\mathrm{P}x)$ by

7: $\mathrm{S}(\mathrm{P}x) = x$

9: $\neg\ \mathrm{S}(\mathrm{P}x) = x \quad \vee \quad \mathrm{Half}(2 \uparrow x) = 2 \uparrow (\mathrm{P}x)$

12: QEA by

10: $\neg\ \mathrm{Half}(2 \uparrow x) = 2 \uparrow (\mathrm{P}x)$

11: $\mathrm{Half}(2 \uparrow x) = 2 \uparrow (\mathrm{P}x)$