Proof of Theorem 236

The theorem to be proved is

q is a power of two $\quad \& \quad q \neq 1 \quad \rightarrow \quad$ Parity $q = 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $\quad$ [[(q) is a power of two] $\quad \& \quad [\neg\, (q) = (1)] \quad \& \quad [\neg\, (\text{Parity}q) = (0)]]$

Special cases of the hypothesis and previous results:

0: q is a power of two $\quad$ from $\quad$ H:q

1: $\neg\, 1 = q$ $\quad$ from $\quad$ H:q

2: $\neg$ Parity$q = 0$ $\quad$ from $\quad$ H:q

3: $\neg\, q$ is a power of two $\quad \vee \quad 2 \uparrow x = q$ $\quad$ from $\quad$ 129$^{\rightarrow}$;q:x

4: $2 \uparrow 0 = 1$ $\quad$ from $\quad$ 126;2;x

5: $0 = x \quad \vee \quad \text{S}(\text{P}x) = x$ $\quad$ from $\quad$ 22;x

6: $\text{Parity}(2 \uparrow (\text{S}(\text{P}x))) = 0$ $\quad$ from $\quad$ 234;Px

Equality substitutions:

7: $\neg\, 2 \uparrow x = q \quad \vee \quad \neg\, \text{Parity}(2 \uparrow x) = 0 \quad \vee \quad \text{Parity}(q) = 0$

8: $\neg\, 2 \uparrow 0 = 1 \quad \vee \quad \neg\, 2 \uparrow 0 = q \quad \vee \quad 1 = q$

9: $\neg\, 0 = x \quad \vee \quad 2 \uparrow 0 = q \quad \vee \quad \neg\, 2 \uparrow x = q$

10: $\neg\, \text{S}(\text{P}x) = x \quad \vee \quad \neg\, \text{Parity}(2 \uparrow (\text{S}(\text{P}x))) = 0 \quad \vee \quad \text{Parity}(2 \uparrow (x)) = 0$

Inferences:

11: $2 \uparrow x = q$ $\quad$ by
0: q is a power of two
3: $\neg\, q$ is a power of two $\quad \vee \quad 2 \uparrow x = q$

12: $\neg\, 2 \uparrow 0 = 1 \quad \vee \quad \neg\, 2 \uparrow 0 = q$ $\quad$ by
1: $\neg\, 1 = q$
8: $\neg\, 2 \uparrow 0 = 1 \quad \vee \quad \neg\, 2 \uparrow 0 = q \quad \vee \quad 1 = q$

13: $\neg\, 2 \uparrow x = q \quad \vee \quad \neg\, \mathrm{Parity}(2 \uparrow x) = 0$ by

2: $\neg\, \mathrm{Parity} q = 0$

7: $\neg\, 2 \uparrow x = q \quad \vee \quad \neg\, \mathrm{Parity}(2 \uparrow x) = 0 \quad \vee \quad \mathrm{Parity} q = 0$

14: $\neg\, 2 \uparrow 0 = q$ by

4: $2 \uparrow 0 = 1$

12: $\neg\, 2 \uparrow 0 = 1 \quad \vee \quad \neg\, 2 \uparrow 0 = q$

15: $\neg\, \mathrm{S}(\mathrm{P}x) = x \quad \vee \quad \mathrm{Parity}(2 \uparrow x) = 0$ by

6: $\mathrm{Parity}(2 \uparrow (\mathrm{S}(\mathrm{P}x))) = 0$

10: $\neg\, \mathrm{S}(\mathrm{P}x) = x \quad \vee \quad \neg\, \mathrm{Parity}(2 \uparrow (\mathrm{S}(\mathrm{P}x))) = 0 \quad \vee \quad \mathrm{Parity}(2 \uparrow x) = 0$

16: $\neg\, 0 = x \quad \vee \quad 2 \uparrow 0 = q$ by

11: $2 \uparrow x = q$

9: $\neg\, 0 = x \quad \vee \quad 2 \uparrow 0 = q \quad \vee \quad \neg\, 2 \uparrow x = q$

17: $\neg\, \mathrm{Parity}(2 \uparrow x) = 0$ by

11: $2 \uparrow x = q$

13: $\neg\, 2 \uparrow x = q \quad \vee \quad \neg\, \mathrm{Parity}(2 \uparrow x) = 0$

18: $\neg\, 0 = x$ by

14: $\neg\, 2 \uparrow 0 = q$

16: $\neg\, 0 = x \quad \vee \quad 2 \uparrow 0 = q$

19: $\neg\, \mathrm{S}(\mathrm{P}x) = x$ by

17: $\neg\, \mathrm{Parity}(2 \uparrow x) = 0$

15: $\neg\, \mathrm{S}(\mathrm{P}x) = x \quad \vee \quad \mathrm{Parity}(2 \uparrow x) = 0$

20: $\mathrm{S}(\mathrm{P}x) = x$ by

18: $\neg\, 0 = x$

5: $0 = x \quad \vee \quad \mathrm{S}(\mathrm{P}x) = x$

21: QEA by

19: $\neg\, \mathrm{S}(\mathrm{P}x) = x$

20: $\mathrm{S}(\mathrm{P}x) = x$