Proof of Theorem 235

The theorem to be proved is

$\mathrm{Half}(2 \uparrow \mathrm{S}x) = 2 \uparrow x$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\, (\mathrm{Half}(2 \uparrow (\mathrm{S}x))) = (2 \uparrow x)]]$

Special cases of the hypothesis and previous results:

0: $\neg\, \mathrm{Half}(2 \uparrow (\mathrm{S}x)) = 2 \uparrow x$ from H:x

1: $\mathrm{S}(\mathrm{S}0) = 2$ from 116

2: $\mathrm{S}0 = 1$ from 115

3: $\mathrm{Parity}(2 \uparrow (\mathrm{S}x)) = 0$ from 234;x

4: $\neg\, \mathrm{Parity}(2 \uparrow (\mathrm{S}x)) = 0 \;\vee\; 2 \cdot (\mathrm{Half}(2 \uparrow (\mathrm{S}x))) = 2 \uparrow (\mathrm{S}x)$ from 224;$2 \uparrow (\mathrm{S}x)$

5: $2 \cdot (2 \uparrow x) = 2 \uparrow (\mathrm{S}x)$ from 126;2;x

6: $\neg\, \mathrm{S}1 = 0$ from 3;1

7: $\neg\, 2 \cdot (\mathrm{Half}(2 \uparrow (\mathrm{S}x))) = 2 \cdot (2 \uparrow x) \;\vee\; 2 = 0 \;\vee\; \mathrm{Half}(2 \uparrow (\mathrm{S}x)) = 2 \uparrow x$ from 198;2;$\mathrm{Half}(2 \uparrow (\mathrm{S}x))$;$2 \uparrow x$

Equality substitutions:

8: $\neg\, \mathrm{S}0 = 1 \;\vee\; \neg\, \mathrm{S}(\mathrm{S}0) = 2 \;\vee\; \mathrm{S}(1) = 2$

9: $\neg\, 2 \cdot (\mathrm{Half}(2 \uparrow (\mathrm{S}x))) = 2 \uparrow (\mathrm{S}x) \;\vee\; 2 \cdot (\mathrm{Half}(2 \uparrow (\mathrm{S}x))) = 2 \cdot (2 \uparrow x) \;\vee\; \neg\, 2 \uparrow (\mathrm{S}x) = 2 \cdot (2 \uparrow x)$

10: $\neg\, 2 = 0 \;\vee\; \neg\, \mathrm{S}1 = 2 \;\vee\; \mathrm{S}1 = 0$

Inferences:

11: $\neg\, 2 \cdot (\mathrm{Half}(2 \uparrow (\mathrm{S}x))) = 2 \cdot (2 \uparrow x) \;\vee\; 2 = 0$ by
0: $\neg\, \mathrm{Half}(2 \uparrow (\mathrm{S}x)) = 2 \uparrow x$
7: $\neg\, 2 \cdot (\mathrm{Half}(2 \uparrow (\mathrm{S}x))) = 2 \cdot (2 \uparrow x) \;\vee\; 2 = 0 \;\vee\; \mathrm{Half}(2 \uparrow (\mathrm{S}x)) = 2 \uparrow x$

12: $\neg\, \mathrm{S}0 = 1 \;\vee\; \mathrm{S}1 = 2$ by
1: $\mathrm{S}(\mathrm{S}0) = 2$
8: $\neg\, \mathrm{S}0 = 1 \;\vee\; \neg\, \mathrm{S}(\mathrm{S}0) = 2 \;\vee\; \mathrm{S}1 = 2$

13: $\mathrm{S}1 = 2$ by

2: $\mathrm{S}0 = 1$

12: $\neg\ \mathrm{S}0 = 1 \quad \vee \quad \mathrm{S}1 = 2$

14: $2 \cdot (\mathrm{Half}(2 \uparrow (\mathrm{S}x))) = 2 \uparrow (\mathrm{S}x)$ by

3: $\mathrm{Parity}(2 \uparrow (\mathrm{S}x)) = 0$

4: $\neg\ \mathrm{Parity}(2 \uparrow (\mathrm{S}x)) = 0 \quad \vee \quad 2 \cdot (\mathrm{Half}(2 \uparrow (\mathrm{S}x))) = 2 \uparrow (\mathrm{S}x)$

15: $\neg\ 2 \cdot (\mathrm{Half}(2 \uparrow (\mathrm{S}x))) = 2 \uparrow (\mathrm{S}x) \quad \vee \quad 2 \cdot (\mathrm{Half}(2 \uparrow (\mathrm{S}x))) = 2 \cdot (2 \uparrow x)$ by

5: $2 \cdot (2 \uparrow x) = 2 \uparrow (\mathrm{S}x)$

9: $\neg\ 2 \cdot (\mathrm{Half}(2 \uparrow (\mathrm{S}x))) = 2 \uparrow (\mathrm{S}x) \quad \vee \quad 2 \cdot (\mathrm{Half}(2 \uparrow (\mathrm{S}x))) = 2 \cdot (2 \uparrow x) \quad \vee \quad \neg\ 2 \cdot (2 \uparrow x) = 2 \uparrow (\mathrm{S}x)$

16: $\neg\ 2 = 0 \quad \vee \quad \neg\ \mathrm{S}1 = 2$ by

6: $\neg\ \mathrm{S}1 = 0$

10: $\neg\ 2 = 0 \quad \vee \quad \neg\ \mathrm{S}1 = 2 \quad \vee \quad \mathrm{S}1 = 0$

17: $\neg\ 2 = 0$ by

13: $\mathrm{S}1 = 2$

16: $\neg\ 2 = 0 \quad \vee \quad \neg\ \mathrm{S}1 = 2$

18: $2 \cdot (\mathrm{Half}(2 \uparrow (\mathrm{S}x))) = 2 \cdot (2 \uparrow x)$ by

14: $2 \cdot (\mathrm{Half}(2 \uparrow (\mathrm{S}x))) = 2 \uparrow (\mathrm{S}x)$

15: $\neg\ 2 \cdot (\mathrm{Half}(2 \uparrow (\mathrm{S}x))) = 2 \uparrow (\mathrm{S}x) \quad \vee \quad 2 \cdot (\mathrm{Half}(2 \uparrow (\mathrm{S}x))) = 2 \cdot (2 \uparrow x)$

19: $\neg\ 2 \cdot (\mathrm{Half}(2 \uparrow (\mathrm{S}x))) = 2 \cdot (2 \uparrow x)$ by

17: $\neg\ 2 = 0$

11: $\neg\ 2 \cdot (\mathrm{Half}(2 \uparrow (\mathrm{S}x))) = 2 \cdot (2 \uparrow x) \quad \vee \quad 2 = 0$

20: QEA by

18: $2 \cdot (\mathrm{Half}(2 \uparrow (\mathrm{S}x))) = 2 \cdot (2 \uparrow x)$

19: $\neg\ 2 \cdot (\mathrm{Half}(2 \uparrow (\mathrm{S}x))) = 2 \cdot (2 \uparrow x)$