Proof of Theorem 234

The theorem to be proved is

$\mathrm{Parity}(2 \uparrow \mathrm{S}x) = 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\ (\mathrm{Parity}(2 \uparrow (\mathrm{S}x))) = (0)]]$

Special cases of the hypothesis and previous results:

0: $\neg\ \mathrm{Parity}(2 \uparrow (\mathrm{S}x)) = 0$ from H:x

1: $2 \cdot (2 \uparrow x) = 2 \uparrow (\mathrm{S}x)$ from 126;2;x

2: $\mathrm{Parity}2 = 0$ from 208

3: $\neg\ \mathrm{Parity}2 = 0 \quad \vee \quad \mathrm{Parity}(2 \cdot (2 \uparrow x)) = 0$ from 211;2;$2 \uparrow x$

Equality substitutions:

4: $\neg\ 2 \cdot (2 \uparrow x) = 2 \uparrow (\mathrm{S}x) \quad \vee \quad \neg\ \mathrm{Parity}(2 \cdot (2 \uparrow x)) = 0 \quad \vee \quad \mathrm{Parity}(2 \uparrow (\mathrm{S}x)) = 0$

Inferences:

5: $\neg\ 2 \cdot (2 \uparrow x) = 2 \uparrow (\mathrm{S}x) \quad \vee \quad \neg\ \mathrm{Parity}(2 \cdot (2 \uparrow x)) = 0$ by
0: $\neg\ \mathrm{Parity}(2 \uparrow (\mathrm{S}x)) = 0$
4: $\neg\ 2 \cdot (2 \uparrow x) = 2 \uparrow (\mathrm{S}x) \quad \vee \quad \neg\ \mathrm{Parity}(2 \cdot (2 \uparrow x)) = 0 \quad \vee \quad \mathrm{Parity}(2 \uparrow (\mathrm{S}x)) = 0$

6: $\neg\ \mathrm{Parity}(2 \cdot (2 \uparrow x)) = 0$ by
1: $2 \cdot (2 \uparrow x) = 2 \uparrow (\mathrm{S}x)$
5: $\neg\ 2 \cdot (2 \uparrow x) = 2 \uparrow (\mathrm{S}x) \quad \vee \quad \neg\ \mathrm{Parity}(2 \cdot (2 \uparrow x)) = 0$

7: $\mathrm{Parity}(2 \cdot (2 \uparrow x)) = 0$ by
2: $\mathrm{Parity}2 = 0$
3: $\neg\ \mathrm{Parity}2 = 0 \quad \vee \quad \mathrm{Parity}(2 \cdot (2 \uparrow x)) = 0$

8: QEA by
6: $\neg\ \mathrm{Parity}(2 \cdot (2 \uparrow x)) = 0$
7: $\mathrm{Parity}(2 \cdot (2 \uparrow x)) = 0$