

## Proof of Theorem 232

The theorem to be proved is

$$2 \cdot \text{Half } x \leq x$$

Suppose the theorem does not hold. Then, with the variables held fixed,

$$(H) \quad [[\neg (2 \cdot (\text{Half}x)) \leq (x)]]$$

### Special cases of the hypothesis and previous results:

- 0:  $\neg 2 \cdot (\text{Half}x) \leq x$  from H: $x$
- 1:  $\neg \text{Parity}x = 0 \vee 2 \cdot (\text{Half}x) = x$  from [224](#); $x$
- 2:  $\neg \text{Parity}x = 1 \vee (2 \cdot (\text{Half}x)) + 1 = x$  from [225](#); $x$
- 3:  $x \leq x$  from [60](#); $x$
- 4:  $2 \cdot (\text{Half}x) \leq (2 \cdot (\text{Half}x)) + 1$  from [71](#); $2 \cdot (\text{Half}x)$ ;1
- 5:  $\text{Parity}x = 0 \vee \text{Parity}x = 1$  from [209](#); $x$

### Equality substitutions:

- 6:  $\neg 2 \cdot (\text{Half}x) = x \vee 2 \cdot (\text{Half}x) \leq x \vee \neg x \leq x$
- 7:  $\neg (2 \cdot (\text{Half}x)) + 1 = x \vee \neg 2 \cdot (\text{Half}x) \leq (2 \cdot (\text{Half}x)) + 1 \vee 2 \cdot (\text{Half}x) \leq x$

### Inferences:

- 8:  $\neg 2 \cdot (\text{Half}x) = x \vee \neg x \leq x$  by
  - 0:  $\neg 2 \cdot (\text{Half}x) \leq x$
  - 6:  $\neg 2 \cdot (\text{Half}x) = x \vee 2 \cdot (\text{Half}x) \leq x \vee \neg x \leq x$
- 9:  $\neg (2 \cdot (\text{Half}x)) + 1 = x \vee \neg 2 \cdot (\text{Half}x) \leq (2 \cdot (\text{Half}x)) + 1$  by
  - 0:  $\neg 2 \cdot (\text{Half}x) \leq x$
  - 7:  $\neg (2 \cdot (\text{Half}x)) + 1 = x \vee \neg 2 \cdot (\text{Half}x) \leq (2 \cdot (\text{Half}x)) + 1 \vee 2 \cdot (\text{Half}x) \leq x$
- 10:  $\neg 2 \cdot (\text{Half}x) = x$  by
  - 3:  $x \leq x$
  - 8:  $\neg 2 \cdot (\text{Half}x) = x \vee \neg x \leq x$

- 11:  $\neg (2 \cdot (\text{Half}x)) + 1 = x$  by  
 4:  $2 \cdot (\text{Half}x) \leq (2 \cdot (\text{Half}x)) + 1$   
 9:  $\neg (2 \cdot (\text{Half}x)) + 1 = x \vee \neg 2 \cdot (\text{Half}x) \leq (2 \cdot (\text{Half}x)) + 1$
- 12:  $\neg \text{Parity}x = 0$  by  
 10:  $\neg 2 \cdot (\text{Half}x) = x$   
 1:  $\neg \text{Parity}x = 0 \vee 2 \cdot (\text{Half}x) = x$
- 13:  $\neg \text{Parity}x = 1$  by  
 11:  $\neg (2 \cdot (\text{Half}x)) + 1 = x$   
 2:  $\neg \text{Parity}x = 1 \vee (2 \cdot (\text{Half}x)) + 1 = x$
- 14:  $\text{Parity}x = 1$  by  
 12:  $\neg \text{Parity}x = 0$   
 5:  $\text{Parity}x = 0 \vee \text{Parity}x = 1$
- 15: *QEA* by  
 13:  $\neg \text{Parity}x = 1$   
 14:  $\text{Parity}x = 1$