Proof of Theorem 230

The theorem to be proved is

$\mathrm{Chop}\,\epsilon = \epsilon$ ⋆

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\ (\mathrm{Chop}\epsilon) = (\epsilon)]]$

Special cases of the hypothesis and previous results:

0: $\neg\ \mathrm{Chop}\epsilon = \epsilon$ from H

1: $\mathrm{P}((\mathrm{Half}(\mathrm{Q}\epsilon)) + (\mathrm{Half}(\mathrm{R}\epsilon))) = \mathrm{Chop}\epsilon$ from 229;ϵ

2: $\mathrm{Q}\epsilon = 1$ from 189

3: $\mathrm{R}\epsilon = 0$ from 189

4: $\epsilon = 0$ from 185

5: $\mathrm{Half}0 = 0$ from 221

6: $\mathrm{Half}1 = 0$ from 221

7: $0 + 0 = 0$ from 12;0

8: $\mathrm{P}0 = 0$ from 16

Equality substitutions:

9: $\neg\ \mathrm{Q}\epsilon = 1 \ \vee\ \neg\ \mathrm{P}((\mathrm{Half}(\mathrm{Q}\epsilon))+(\mathrm{Half}(\mathrm{R}\epsilon))) = \mathrm{Chop}\epsilon \ \vee\ \mathrm{P}((\mathrm{Half}(1))+(\mathrm{Half}(\mathrm{R}\epsilon))) = \mathrm{Chop}\epsilon$

10: $\neg\ \mathrm{R}\epsilon = 0 \ \vee\ \neg\ \mathrm{P}((\mathrm{Half}1)+(\mathrm{Half}(\mathrm{R}\epsilon))) = \mathrm{Chop}\epsilon \ \vee\ \mathrm{P}((\mathrm{Half}1)+(\mathrm{Half}(0))) = \mathrm{Chop}\epsilon$

11: $\neg\ \mathrm{Half}0 = 0 \ \vee\ \neg\ \mathrm{P}((\mathrm{Half}1) + (\mathrm{Half}0)) = \mathrm{Chop}\epsilon \ \vee\ \mathrm{P}((\mathrm{Half}1) + (0)) = \mathrm{Chop}\epsilon$

12: $\neg\ \mathrm{Half}1 = 0 \ \vee\ \mathrm{P}((\mathrm{Half}1) + 0) = \mathrm{P}0 \ \vee\ \neg\ \mathrm{P}((0) + 0) = \mathrm{P}0$

13: $\neg\ 0 + 0 = 0 \ \vee\ \mathrm{P}(0 + 0) = \mathrm{P}0 \ \vee\ \neg\ \mathrm{P}(0) = \mathrm{P}0$

14: $\neg\ \mathrm{P}0 = 0 \ \vee\ \neg\ \mathrm{Chop}\epsilon = \mathrm{P}0 \ \vee\ \mathrm{Chop}\epsilon = 0$

15: $\neg\ \mathrm{P}((\mathrm{Half}1) + 0) = \mathrm{Chop}\epsilon \ \vee\ \neg\ \mathrm{P}((\mathrm{Half}1) + 0) = \mathrm{P}0 \ \vee\ \mathrm{Chop}\epsilon = \mathrm{P}0$

16: $\neg\ \mathrm{Chop}\epsilon = 0 \ \vee\ \mathrm{Chop}\epsilon = \epsilon \ \vee\ \neg\ 0 = \epsilon$

Inferences:

17: $\neg$ Chop$\epsilon = 0 \quad \vee \quad \neg\, \epsilon = 0$ by

0: $\neg$ Chop$\epsilon = \epsilon$

16: $\neg$ Chop$\epsilon = 0 \quad \vee \quad$ Chop$\epsilon = \epsilon \quad \vee \quad \neg\, \epsilon = 0$

18: $\neg$ Q$\epsilon = 1 \quad \vee \quad$ P((Half1) + (Half(Rϵ))) = Chopϵ by

1: P((Half(Qϵ)) + (Half(Rϵ))) = Chopϵ

9: $\neg$ Q$\epsilon = 1 \quad \vee \quad \neg$ P((Half(Qϵ))+(Half(Rϵ))) = Chop$\epsilon \quad \vee \quad$ P((Half1)+(Half(Rϵ))) = Chopϵ

19: P((Half1) + (Half(Rϵ))) = Chopϵ by

2: Q$\epsilon = 1$

18: $\neg$ Q$\epsilon = 1 \quad \vee \quad$ P((Half1) + (Half(Rϵ))) = Chopϵ

20: $\neg$ P((Half1) + (Half(Rϵ))) = Chop$\epsilon \quad \vee \quad$ P((Half1) + (Half0)) = Chopϵ by

3: R$\epsilon = 0$

10: $\neg$ R$\epsilon = 0 \quad \vee \quad \neg$ P((Half1) + (Half(Rϵ))) = Chop$\epsilon \quad \vee \quad$ P((Half1) + (Half0)) = Chopϵ

21: $\neg$ Chop$\epsilon = 0$ by

4: $\epsilon = 0$

17: $\neg$ Chop$\epsilon = 0 \quad \vee \quad \neg\, \epsilon = 0$

22: $\neg$ P((Half1) + (Half0)) = Chop$\epsilon \quad \vee \quad$ P((Half1) + 0) = Chopϵ by

5: Half0 = 0

11: $\neg$ Half0 = 0 $\quad \vee \quad \neg$ P((Half1) + (Half0)) = Chop$\epsilon \quad \vee \quad$ P((Half1) + 0) = Chopϵ

23: P((Half1) + 0) = P0 $\quad \vee \quad \neg$ P(0 + 0) = P0 by

6: Half1 = 0

12: $\neg$ Half1 = 0 $\quad \vee \quad$ P((Half1) + 0) = P0 $\quad \vee \quad \neg$ P(0 + 0) = P0

24: P(0 + 0) = P0 by

7: $0 + 0 = 0$

13: $\neg\, 0 + 0 = 0 \quad \vee \quad$ P(0 + 0) = P0

25: $\neg$ Chopϵ = P0 $\quad \vee \quad$ Chop$\epsilon = 0$ by

8: P0 = 0

14: $\neg$ P0 = 0 $\quad \vee \quad \neg$ Chopϵ = P0 $\quad \vee \quad$ Chop$\epsilon = 0$

26: $\mathrm{P}((\mathrm{Half}1) + (\mathrm{Half}0)) = \mathrm{Chop}\epsilon$ by

19: $\mathrm{P}((\mathrm{Half}1) + (\mathrm{Half}(\mathrm{R}\epsilon))) = \mathrm{Chop}\epsilon$

20: $\neg\, \mathrm{P}((\mathrm{Half}1) + (\mathrm{Half}(\mathrm{R}\epsilon))) = \mathrm{Chop}\epsilon \quad \vee \quad \mathrm{P}((\mathrm{Half}1) + (\mathrm{Half}0)) = \mathrm{Chop}\epsilon$

27: $\neg\, \mathrm{Chop}\epsilon = \mathrm{P}0$ by

21: $\neg\, \mathrm{Chop}\epsilon = 0$

25: $\neg\, \mathrm{Chop}\epsilon = \mathrm{P}0 \quad \vee \quad \mathrm{Chop}\epsilon = 0$

28: $\mathrm{P}((\mathrm{Half}1) + 0) = \mathrm{P}0$ by

24: $\mathrm{P}(0 + 0) = \mathrm{P}0$

23: $\mathrm{P}((\mathrm{Half}1) + 0) = \mathrm{P}0 \quad \vee \quad \neg\, \mathrm{P}(0 + 0) = \mathrm{P}0$

29: $\mathrm{P}((\mathrm{Half}1) + 0) = \mathrm{Chop}\epsilon$ by

26: $\mathrm{P}((\mathrm{Half}1) + (\mathrm{Half}0)) = \mathrm{Chop}\epsilon$

22: $\neg\, \mathrm{P}((\mathrm{Half}1) + (\mathrm{Half}0)) = \mathrm{Chop}\epsilon \quad \vee \quad \mathrm{P}((\mathrm{Half}1) + 0) = \mathrm{Chop}\epsilon$

30: $\neg\, \mathrm{P}((\mathrm{Half}1) + 0) = \mathrm{Chop}\epsilon \quad \vee \quad \neg\, \mathrm{P}((\mathrm{Half}1) + 0) = \mathrm{P}0$ by

27: $\neg\, \mathrm{Chop}\epsilon = \mathrm{P}0$

15: $\neg\, \mathrm{P}((\mathrm{Half}1) + 0) = \mathrm{Chop}\epsilon \quad \vee \quad \neg\, \mathrm{P}((\mathrm{Half}1) + 0) = \mathrm{P}0 \quad \vee \quad \mathrm{Chop}\epsilon = \mathrm{P}0$

31: $\neg\, \mathrm{P}((\mathrm{Half}1) + 0) = \mathrm{Chop}\epsilon$ by

28: $\mathrm{P}((\mathrm{Half}1) + 0) = \mathrm{P}0$

30: $\neg\, \mathrm{P}((\mathrm{Half}1) + 0) = \mathrm{Chop}\epsilon \quad \vee \quad \neg\, \mathrm{P}((\mathrm{Half}1) + 0) = \mathrm{P}0$

32: QEA by

29: $\mathrm{P}((\mathrm{Half}1) + 0) = \mathrm{Chop}\epsilon$

31: $\neg\, \mathrm{P}((\mathrm{Half}1) + 0) = \mathrm{Chop}\epsilon$