Proof of Theorem 228

The theorem to be proved is

$x \neq \epsilon \quad \rightarrow \quad \text{Half } \mathrm{Q}x + \text{Half } \mathrm{R}x \neq 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\ (x) = (\epsilon)] \quad \& \quad [((\text{Half}(\mathrm{Q}x)) + (\text{Half}(\mathrm{R}x))) = (0)]]$

Special cases of the hypothesis and previous results:

0: $\neg\ \epsilon = x$ from H:x

1: $(\text{Half}(\mathrm{Q}x)) + (\text{Half}(\mathrm{R}x)) = 0$ from H:x

2: $\neg\ (\text{Half}(\mathrm{Q}x)) + (\text{Half}(\mathrm{R}x)) = 0 \quad \vee \quad \text{Half}(\mathrm{Q}x) = 0$ from 15;Half(Qx);Half(Rx)

3: $\neg\ \text{Half}(\mathrm{Q}x) = 0 \quad \vee \quad \mathrm{Q}x = 0 \quad \vee \quad \mathrm{Q}x = 1$ from 227;Qx

4: $\neg\ \mathrm{Q}x = 0$ from 178;x

5: $\neg\ \mathrm{Q}x = 1 \quad \vee \quad \epsilon = x$ from 203;x

Inferences:

6: $\neg\ \mathrm{Q}x = 1$ by

0: $\neg\ \epsilon = x$

5: $\neg\ \mathrm{Q}x = 1 \quad \vee \quad \epsilon = x$

7: $\text{Half}(\mathrm{Q}x) = 0$ by

1: $(\text{Half}(\mathrm{Q}x)) + (\text{Half}(\mathrm{R}x)) = 0$

2: $\neg\ (\text{Half}(\mathrm{Q}x)) + (\text{Half}(\mathrm{R}x)) = 0 \quad \vee \quad \text{Half}(\mathrm{Q}x) = 0$

8: $\neg\ \text{Half}(\mathrm{Q}x) = 0 \quad \vee \quad \mathrm{Q}x = 1$ by

4: $\neg\ \mathrm{Q}x = 0$

3: $\neg\ \text{Half}(\mathrm{Q}x) = 0 \quad \vee \quad \mathrm{Q}x = 0 \quad \vee \quad \mathrm{Q}x = 1$

9: $\neg\ \text{Half}(\mathrm{Q}x) = 0$ by

6: $\neg\ \mathrm{Q}x = 1$

8: $\neg\ \text{Half}(\mathrm{Q}x) = 0 \quad \vee \quad \mathrm{Q}x = 1$

10: QEA by

7: $\text{Half}(\mathrm{Q}x) = 0$

9: $\neg\ \text{Half}(\mathrm{Q}x) = 0$